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THE READJUSTMENT OF THE INDIAN TRIANGULATION

BY

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INTRODUCTION

The geodetic triangulation of India was first adjusted to form a self-consistent whole in about 1880, and that of Burma in 1916. It was not possible to include any Laplace stations in these adjustments, and it is now known that the adjusted triangulation errs considerably in azimuth in Burma and southern India. Additional series and base-lines which have been observed since the date of the first adjustments also call for inclusion, and a readjustment has now been undertaken which makes use of all the data at present available. So far as most of India is concerned, it is unlikely that future field work will enforce any change in the results now obtained, but changes will be necessary in Assam and Burma, where the figures now given are still only preliminary.

The method used for the readjustment is described in Chapter I, with explanatory details in Appendices I to IV. The results are given in Plate VII, in a form suitable for correcting the geodetic positions of the astronomical stations used for finding the form of the geoid. Table 1 A gives revised values of scale, azimuth and position at 65 stations in India, on which the intermediate primary triangulation can be adjusted series by series, when this adjustment is required as a basis for topography. Table 1 B gives similar (preliminary) data for Assam and Burma. Least squares have been avoided in the readjustment.

The accuracy of the readjusted triangulation is dealt with in Chapter II and Appendices V to X. Plate IX shows the probable errors of the primary triangulation relative to the origin, and Plate X shows the probable errors of the secondary triangulation relative to the primary. Plate IX shows that the length and breadth of the country have been measured with probable errors of about 1 in 500,000 or 20 feet in 2,000 miles. A rigorous method of determining the probable errors of an adjusted triangulation system was given by Dr. J. de Graaff Hunter in Professional Paper No. 16 (1918), pages 89 to 161, but it is not simple to apply. The treatment now proposed is less rigorous, but results should be correct within 20%.

The Indian triangulation is computed on Everest's spheroid, which is known to differ seriously from any spheroid which at all accurately fits the shape of the earth. Chapter III and Appendices XI to XIV give formulæ for converting data from one spheroid to another, and Plates XI and XII provide approximate means for converting from Everest's spheroid to the International spheroid as at present best fitted to India. This subject also was dealt with by Dr. J. de Graaff Hunter in Professional Paper No. 16, pages 1 to 88, where he concluded that the most correct method was that described as calculation along geodesics. This conclusion was correct for the triangulation as it then was, but simpler treatment is possible now that it has been adequately adjusted on base-lines and Laplace stations.

Chapter IV outlines the work necessary for the completion of India's geodetic triangulation, and also discusses the advisability of generally adopting the new adjustment and a new spheroid. It is concluded that use of the new adjustment should be confined to scientific purposes, as opposed to topographical, until a new spheroid is adopted, and that the general adoption of a new spheroid should be postponed until some distant date when other great changes, such as the introduction of metre contours or an increase in the normal scale of survey, will minimize the inconvenience arising from this change.

The four chapters have been kept as brief as possible in order that they may be reasonably readable. Explanatory details and proofs have been placed in appendices. The latter need only be read if some point in the chapters appears unconvincing, or if it is desired to apply the methods used to further work in India or elsewhere.

The readjustment of the triangulation, as far as it has been taken, has involved no heavy computation, and all that has been necessary has been done by slide-rule. The duplicate copy of the computations and of the numerous extracts from original records has been undertaken by Mr. Harendra Chandra Deva B.A., Head Computer, to whose care the elimination of many errors is due.

CHAPTER I

ADJUSTMENT OF THE TRIANGULATION

1. **Previous adjustments.**—The geodetic triangulation of India takes the form of a "grid-iron", or system of series intersecting each other roughly at right-angles (see Frontispiece). By about 1880 the triangulation of most of India was sufficiently near completion for its simultaneous adjustment to be undertaken. The details and results are given in G. T. Vols.* I to IVA, VI to VIII and XII to XIV. Except for a constant change of $-2' 27'' \cdot 18$ in longitude, this adjustment has remained the basis of all Indian triangulation and mapping ever since, and is likely to continue to do so for some time. Its results are now incorporated in the "Triangulation pamphlets".†

By 1916 the triangulation of Burma had reached a stage at which it became necessary to make a preliminary adjustment, although it was recognized that a final adjustment would be necessary after a comparatively short time, as much work still remained to be done. This 1916 adjustment is the basis of the triangulation pamphlets of Burma.

2. **New triangulation series.**—Since the time of these first adjustments, a number of new series have been observed or for the first time included in completed circuits, of which the most important from the present point of view are:—

In India proper‡

No. 62	Jodhpur meridional ...	... 1873–76
" 64	Eastern Sind meridional ...	... 1876–81
" 84	Villupuram series (secondary)	... 1911–12
" 85	Sambalpur meridional ...	... 1911–14
" 92	Middle Godāvari (secondary)...	... 1914–15
" 96	Madura series (secondary) ...	... 1916–17

In Baluchistān

No. 69	Makrān longitudinal ...	... 1895–97
" 74	Kalāt longitudinal ...	... 1904–08
" 76	North Baluchistān series ...	... 1908–10
" 107	Dālbandin series ...	... 1931–32

In Assam and Burma

No. 72	Great Salween series ...	1900–11 & 1929–31
" 80	Upper Irrawaddy series ...	... 1909–11
" 91	Nāga Hills series (secondary)	... 1913–14
" 103	Chittagong series ...	... 1928–30
" 104	Mong Hsat series (re-observed)	... 1929–31
" 34	Assam longitudinal (extended)	... 1934–36
" 66	Mandalay meridional (extended)	... 1936–37

These and all other new series have from time to time been fitted between the positions of their terminal stations as given by the 1880 & 1916 (Burma) adjustments, and the results so obtained have been published in the triangulation pamphlets.

* "Account of the operations of the Great Trigonometrical Survey of India" Vols. I to XIX.
† A series of about 500 pamphlets, published by degree squares, which give the geodetic triangulation data of India and Burma.
‡ i.e., within the circuits of the 1880 adjustment.

3. New base-lines and Laplace stations.—The 1880 adjustment of India depended on 10 base-lines evenly distributed throughout the area, while the preliminary 1916 adjustment of Burma depended on the computed side lengths of the weak Indian triangulation in Bengal and Assam. In recent years seven new base-lines have been measured as follows :—

In India proper	Poona	...	1933–34
In Baluchistān	Padag	...	1933–34
In Assam and Burma	Kēng Tung	...	1930–31
	Mergui	...	1932–33*
	Amherst	...	1932–33
	Kalemyo	...	1932–33
	Nāmtiāli	...	1933–34

The values accepted for the old base-lines also require some change, since they have been reduced to sea-level through their height above the geoid, while reduction through height above the spheroid is correct. See para 22 and Appendix I.

No use was made of Laplace stations in the previous adjustments, and this constitutes their weakest feature. Frequent azimuth observations have always been made, but it has been realized that they cannot be used unless associated with longitude stations. By 1906 the necessary data for 16 Laplace stations were available, and this figure has now been increased to 43 stations or groups of stations. For details see Appendix II.

4. Object of readjustment.—Initially there are two reasons for carrying out the simultaneous adjustment of any system of triangulation, viz :—

(*a*) To obtain the most probable positions for all stations of the triangulation.

(*b*) To disperse circuit closing errors, and so to make the whole self-consistent.

Of these two objects the latter is generally the most important. The present system of maintaining the old adjustment, fitting new series into place within it, and ignoring Laplace stations and new base-lines, does not seriously violate self-consistency, and in this important aspect it is satisfactory, even though it does result in the accepted latitudes and longitudes differing from their most probable values by as much as one second. The undertaking of a new adjustment therefore requires justification. Its objects are :—

(*a*) To obtain the most probable latitudes and longitudes for astronomical stations used for the study of the deviation of the vertical and the form of the geoid. See para 10.

(*b*) To determine how much the old adjustment is in error, so that an opinion can be formed on the desirability of adopting the new adjustment or of retaining the old. See Plate VII.

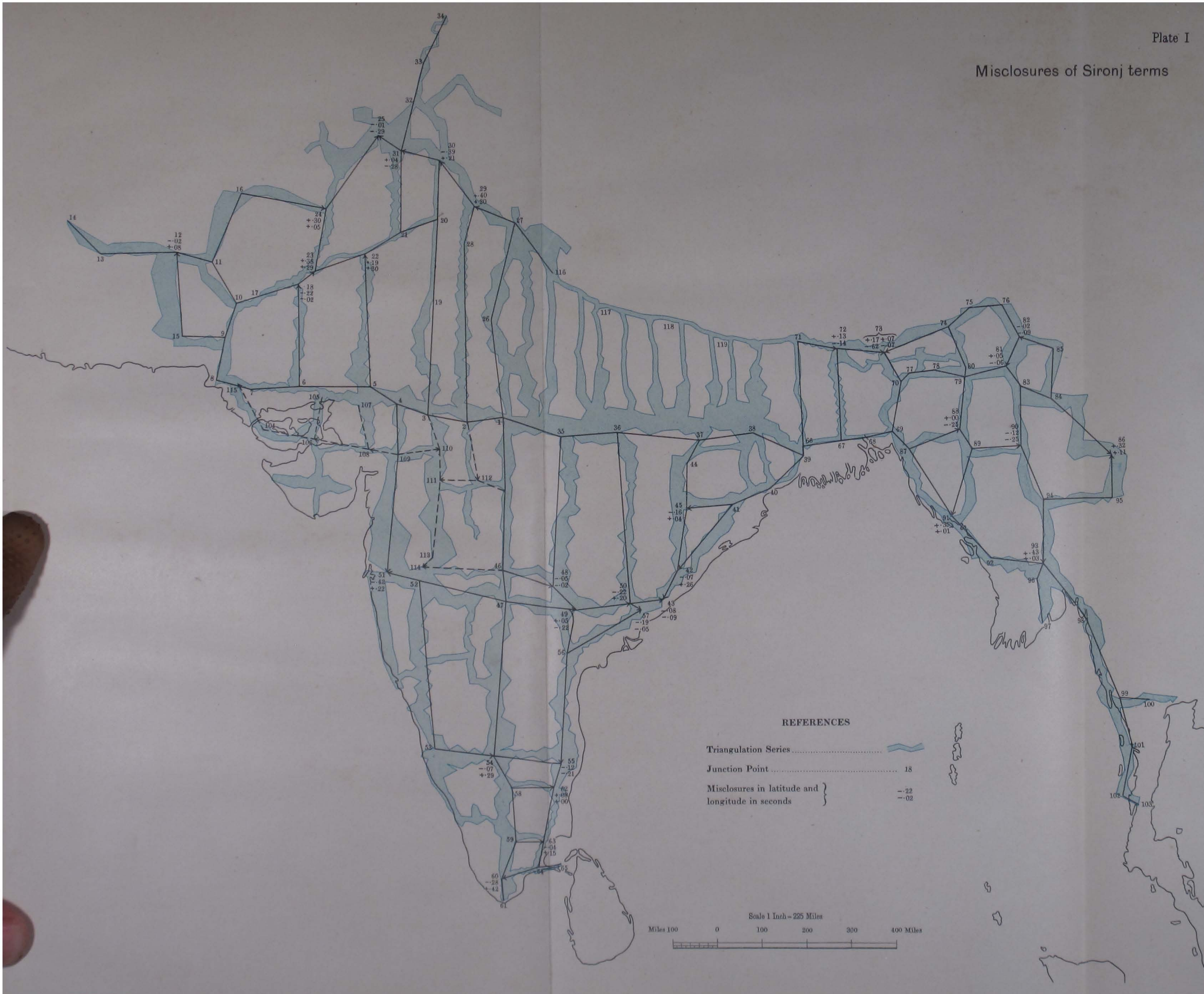
(*c*) To ascertain the probable accuracy of existing series, whereby the necessity for further work can be judged. This is discussed in Chapter II.

5. Method used.—Plate I shows lines which have been chosen to generalize each primary series and a few secondary series so placed as to add strength to the primary system. The lines pass through the stations at which the computations of the series concerned have been started and closed.

* The Mergui base-line was originally measured in 1881–82. It was considered doubtful and ignored in the 1916 adjustment. Remeasurement in 1932–33 differed from the old value by 1 in 170,000.

Plate I

Misclosures of Sironj terms



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At various places one or more of the lines terminate in "circuit points" (marked by arrows on Plate I), which are so arranged that (except at these points themselves) every point in the triangulation can be reached from Point 1, the origin of the survey, by one route only. For instance Point 16 may be reached by the route 1 – 5 – 8 – 10 – 11 – 16. The two routes by which a circuit point can be reached from the origin are referred to as the right-hand and left-hand branches, with reference to their relative positions as the circuit point is approached from the origin.

The original observed angles modified only by the figural adjustments* have then been used to compute the triangulation from the origin along the selected routes, producing preliminary values of scale, azimuth, latitude and longitude which are described as "Sironj" terms†. Most of this computation has already been carried out in G. T. Vols. II to XIV, and little work has been involved now. See Appendix III, para 3. Misclosures occur at all circuit points, and Sironj values of scale and azimuth will not in general agree with the observed values at other base-lines or Laplace stations. The object of the adjustment is to disperse these discrepancies in the most reasonable manner. They are shown in Plates I, II and III.

The errors of scale as revealed at base-lines, and the scale discrepancies at circuit points have first been distributed in what is apparently the most probable manner taking into account the relative strength of the different series, but ignoring misclosures of latitude and longitude. The result is shown in Plate II in the form of corrections applicable to Sironj-terms log sides. Plate III similarly shows the most probable distribution of the azimuth errors and discrepancies.

Accepting the corrections of Plates II and III the latitudes and longitudes of all "junction points"‡ have been recomputed, producing what has been called adjustment A. Misclosures of latitude and longitude of course remain at all circuit points, and are shown in Plate IV. If base-lines and Laplace stations were very numerous, the new misclosures in latitude and longitude would be materially smaller than those of the original Sironj terms. Actually they are considerably decreased in 9 cases and considerably increased in 3, while the remaining 20 misclosures are evenly divided between small increases and small decreases.

The corrections to Sironj-terms scale and azimuth have next been redistributed between control points (base-lines and Laplace stations) by trial and error, in such a way as to secure zero circuit misclosures in latitude and longitude. Distribution by trial and error is of course an arbitrary proceeding to some extent, but the closure of the larger circuit errors imposes some considerable strain on the series involved, and the equitable dispersal of this strain over the different series, in proportion to their presumed inherent liability to error, is not a matter in which any very wide choice is possible. See also para 8 and Appendix III, para 5.

The result of the above process is to give the scale, azimuth, latitude and longitude at all junction points of series. See Tables 1 A & 1 B. For a complete readjustment (when required), which will provide revised consistent data at every triangulation station, individual series will be adjusted on to these terminal values by the method at present employed for fitting new series into the old adjustment. See Appendix III, para 6. The desirability of undertaking this work at present is discussed in Chapter IV.

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* Distribution of triangular errors, grinding of quadrilaterals etc.

† i.e., in terms of the base-line at Sironj and of the accepted fundamental azimuth, latitude and longitude at the near-by Kaliānpur H.S., which is the origin.

‡ "Junction points" are all the points numbered on Plate I.

The above is an outline of the procedure which has been followed. Fuller details are given in Appendix III. It is now necessary to justify its use, explaining in particular why use has not been made of :—

(*a*) The method previously used in India.

or (*b*) The Bowie method, recently used in the United States.

or (*c*) Any other new method involving least squares in preference to trial and error.

6. The Indian method of 1880.—As in para 5, the observed angles were first corrected for the figural adjustments, and a selected chain of single triangles was computed in Sironj terms through each series along routes such as those shown in Plate I, producing circuit misclosures and discrepancies at measured base-lines. Weights were separately allotted to two angles of each triangle, and the most probable corrections to all these angles were then determined by least squares, the equations of condition being given by the base-lines and circuit misclosures. To avoid an impossible number of simultaneous normal equations, the series were grouped into five main sections which were adjusted in order, the adjusted values of the first being used as the basis of the second, and so on, as follows :—

NW. Quadrilateral	with	23	conditions	and	1100	unknowns
SE. Quadrilateral	,,	15	,,	,,	554	,,
South Trigon	,,	22	,,	,,	606	,,
SW. Quadrilateral	,,	24	,,	,,	344	,,
NE. Quadrilateral	,,	49	,,	,,	1146	,,

Owing to the comparative weakness of the last two sections this subdivision of the work was of very slight disadvantage.

The errors of two angles of each triangle being determined, it is a straightforward though laborious process to determine self-consistent revised values for the scale, azimuth, latitude and longitude at each station.

If this method had been adopted for the present readjustment, the number of conditions and unknowns in each section would have been as follows :—

NW. Quadrilateral	: 48	conditions	and	about	1420	unknowns
SE. Quadrilateral	: 30	,,	,,	,,	660	,,
South Trigon	: 35	,,	,,	,,	660	,,
Assam–Burma	: 58	,,	,,	,,	1000	,,

The SW. and NE. Quadrilaterals are not considered as they are of secondary accuracy.

It would be possible to solve these equations if it was indispensable, but to do so would be very laborious. This system, whereby the errors determined are those of so many different angles, involves especially great labour in the formation of the normal equations and in the final determination of the latitudes and longitudes, since it is an extreme example of working up from the part to the whole, and the angular corrections have to be determined to 3 decimals of a second in order to secure consistent results at the circuit closing points.

For these reasons, and those stated in para 8, it has been thought better to adopt some simpler method.

7. The Bowie method.—This method is applicable to a triangulation system which is well supplied with base-lines and Laplace stations. At each point where series branch or intersect the scale and azimuth of one side is first decided on and held fixed thereafter. In the United States the necessary control is either located at all series junctions, or sufficiently close for this to be possible.

The series are then computed using these values of scale and azimuth as a basis, and also accepting the results of any intermediate base-lines and Laplace stations. Circuit errors in latitude and longitude result, and least-square solutions are then made for the latitude and longitude (separately) to be accepted at each series junction. Each series is then finally adjusted between these values of its terminal junction points.

This system is an excellent one if sufficient base and Laplace control exists. In India there is not nearly sufficient, and the Bowie method cannot be used. The scale and azimuth at most series junctions are too indeterminate to be finally assignable in the early stages of the adjustment, and the essence of the method now employed is that circuit errors in latitude and longitude must be dispersed by varying the preliminary values of scale and azimuth allotted to these points in adjustment A.

8. Least squares.—The merits of least squares have often been the subject of controversy. It may be agreed that this method gives a solution which is as probable as any other, and more probable than most, provided :—

(*a*) Weights are correctly assigned.

(*b*) No mistakes are made.

With regard to weights, it must be admitted that the relative weights of the angles of different triangulation series can only be determined within very broad limits. Examples of this are given in Appendix VI, where (e.g.) the observed angles of the East Calcutta series are now assigned a probable error of $1''\cdot0$, compared with an average of $0''\cdot23$ in the 1880 adjustment. The change of weight is nearly 20-fold. This is an extreme case, but it does not stand alone. In the presence of such doubts in the data, common-sense and a slide rule appear to be good substitutes for elaborate computations.

With regard to the absence of mistakes, it is possible to impose checks which will prove the absence of arithmetical blunders in the solution of the equations, but that does not prove that everything is correct. A considerable amount of skill and experience is needed when forming large numbers of simultaneous equations, if instability is to be avoided. Instability, if it occurs, may be so bad as to be obvious, in which case no error results but the work has to be started again. A lesser amount of instability, on the other hand, perhaps not bad enough to be obvious, may produce results which are appreciably wrong in spite of the proving of the usual arithmetical checks. Again, there may be a blunder in the data, such as that described in Appendix III, para 8. The effects of such a blunder are completely obscured in the arithmetic of a least-square solution, and do not affect the arithmetical checks, while in a common-sense adjustment it is likely to make its presence felt.

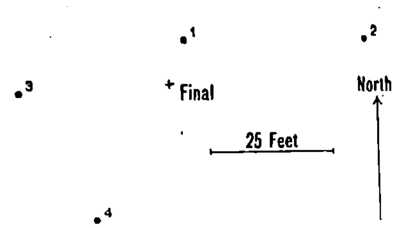
Plate VI shows the positions finally accepted for series junctions, relative to the discrepant positions given by Adjustment A. Giving attention to the larger discrepancies such as those at points 23 and 24, it is seen that the positions accepted are reasonable. Consider the closure at point 23, for instance, in more detail. The marginal figure compares the final position with that obtained by following different routes with the scales and azimuths of Adjustment A :—

(1) 1 – 5 – 22 – 23

(2) 1 – 3 – 20 – 22 – 23

(3) 1 – 5 – 8 – 10 – 23

(4) 1 – 5 – 6 – 18 – 23



The position finally arrived at by the adjustment is well in accord with the lengths and strengths of the different routes*. Remembering that the weights of the different routes may have been wrongly allotted by several hundred per cent, a final position 10 feet different could hardly be described as wrong, although it would look less probable. If a least-square solution gave a position differing by this amount, it could not be described as a better one, while if accepting the same weights it gave a solution differing by (say) 20 feet the only proper course would be to seek for the error in the least square solution until it was found. Under these circumstances, it is thought that the substitution of trial and error for least squares has saved much labour, avoided blunders, and sacrificed no accuracy.

An advantage of the method now used, which it shares with the Bowie method, is that for the final and inevitably laborious process of obtaining self-consistent values of scale, azimuth, latitude and longitude at every triangulation station (when required), the computations will be divided up into about 100 completely independent pieces of work. There will be no practical limit to the number of computers who can be employed, and none will be dependent on the speed or accuracy of any other.

It must be said that least squares are not to be regarded as useless for all purposes. For instance, the observation of redundant rays and the consequent formation of fairly complicated figures† is advocated. It is thought that such figures should be ground by least squares, for the circumstances differ from those of the simultaneous adjustment of a whole system. The number of equations can be kept low : the relative weights of the angles of a single figure can be reasonably accurately determined : and the number of conditions may perhaps be as much as half the number of observed angles, in which case a good adjustment can add considerably to the strength of the angles.

9. **Results of the readjustment.**—Table 1 A for India, and Table 1 B for Assam–Burma, show the scale, azimuth, latitude and longitude at each series junction point as determined by

(*a*) Sironj terms.

(*b*) Adjustment A.

(*c*) 1880 and 1916 (Burma) adjustments.

(*d*) 1937 Adjustment.

In India the 1937 figures are final‡ (for Everest's spheroid) except at certain junction points marked by an asterisk, which will be treated as

* The process by which the final position was arrived at is described in Appendix III, and has no connection with the marginal figure. The fact that the position appears reasonable in the figure is thus an independent check on the adjustment.

† Up to a maximum of about 12 conditions.

‡ If, as is probable, the detailed adjustment of intermediate stations is postponed for some years, it may be desirable to reconsider parts of the present adjustment, and to incorporate the results of later work, such as items (3) and (5) of para 26.

detailed in Appendix III, the last part of para 6. In Burma these figures are provisional only, since Burma will have to be readjusted in the same way, when work now in hand has been completed in about 1941. See Chapter IV, para 28. The changes in latitude and longitude between the old and new adjustments are shown in Plate VII. They are very small (unplottable at the scale of one inch to a mile) except :—

(*a*) South of Bangalore (latitude 13°) where changes of longitude of up to 90 feet occur.

(*b*) In Burma, where changes of over 100 feet occur in both latitude and longitude. South of Amherst, latitude 16°, there are changes in longitude of over 200 feet, but this triangulation is not included in Table 1 B, as it requires to be adjusted simultaneously with that of Siam. See para 28. The corrections shown in Plate VII are, however, approximately correct.

The comparatively large changes in both these areas principally arise from the introduction of Laplace stations. These show that the triangulation as previously adjusted without Laplace stations had developed about 8 seconds of azimuth error at Cape Comorin, and up to 12 seconds in Burma.

10. Astronomical stations.—Revised values of the geodetic latitude and longitude of astronomical stations, where the deviation of the vertical has been observed, can be obtained to the nearest $0''\cdot 1$ by applying the corrections shown in Plate VII to the values at present accepted. The correction to the deviation of the vertical is the same with opposite sign, and after multiplication by $\cos \phi$ in the case of deviations deduced from longitude observations.

Revised geodetic azimuths for determination of the deviation of the vertical at azimuth stations, can similarly be deduced by interpolating corrections from Plate V*, and applying them to Sironj-terms azimuths. The changes from the values at present accepted will not necessarily be large, as a preliminary distribution of the Laplace closures then available was made in Professional Paper No. 16†, and was used to give improved geodetic azimuths for this purpose.

11. Adjustment of secondary triangulation.—When a complete detailed readjustment of secondary series is required, isolated secondary series will be adjusted between the new values of their primary terminal stations in the same way as primary series are adjusted between the values given in Tables 1 A and 1 B. This simple treatment will not be possible for the adjustment of the networks of secondary series forming the North-east and South-west Quadrilaterals, and details of the treatment proposed are given in Appendix IV.

* Ignoring the entries in square brackets. These additional corrections serve to close the circuits, but are not likely to increase the accuracy of the azimuths.

† J. de Graaff Hunter, 1918, pages 165–169.

TABLE 1A

India

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
Station 1. Kaliānpur–Sūrantāl					
Sironj terms	...	4·643 9263	190 27 05·1	24 07 11·26	77 39 17·57
Adjustment A	...	9263	05·1	11·26	17·57
Published values	...	9262	05·1	11·26	17·57
1937 Adjustment	...	9263	05·1	11·26	17·57
Station 2. Rangāon–Dāwa					
Sironj terms	...	4·921 9081	292 30 02·3	23 54 35·44	76 23 06·66
Adjustment A	...	9081	02·3	35·44	06·66
Published values	...	9084	02·4	35·44	06·66
1937 Adjustment*	...	9081	02·3	35·44	06·66
Station 3. Bālagarra–Būda					
Sironj terms	...	4·796 0908	248 10 32·4	24 10 21·89	74 57 48·66
Adjustment A	...	0898	32·4	21·89	48·67
Published values	...	0898	32·6	21·90	48·66
1937 Adjustment	...	0878	32·4	21·89	48·68
Station 4. Lakarwas–Tāna					
Sironj terms	...	5·138 3161	240 10 35·6	24 31 47·98	73 49 43·20
Adjustment A	...	3159	35·6	47·98	43·21
Published values	...	3142	36·4	47·99	43·23
1937 Adjustment*	...	3109	34·6	47·96	43·25
Station 5. Bonik–Sūnda					
Sironj terms	...	5·254 1480	55 04 14·8	25 03 51·47	72 51 54·63
Adjustment A	...	1485	14·7	51·47	54·64
Published values	...	1461	15·7	51·50	54·67
1937 Adjustment	...	1465	14·7	51·42	54·71
Station 6. Rojhra–Sandohar					
Sironj terms	...	4·961 3161	111 55 36·2	24 57 26·22	70 14 17·83
Adjustment A	...	3181	35·8	26·21	17·81
Published values	...	3162	37·1	26·28	17·90
1937 Adjustment	...	3201	35·8	26·18	17·89
Station 7. Chūtli–Kanād					
Sironj terms	...	4·767 3527	173 26 15·1	24 46 19·58	68 23 40·82
Adjustment A	...	3577	14·4	19·55	40·75
Published values	...	3567	15·6	19·67	40·86
1937 Adjustment	...	3554	14·4	19·49	40·82
Station 8. Magar Pīr–Maio					
Sironj terms	...	4·868 3327	196 55 56·9	24 59 16·05	67 01 28·26
Adjustment A	...	3396	54·4	15·99	28·12
Published values	...	3394	57·1	16·15	28·25
1937 Adjustment	...	3376	54·5	15·93	28·22
Station 9. Sūlimāni–Andar					
Sironj terms	...	5·209 0117	1 07 19·4	26 28 04·35	67 12 45·32
Adjustment A	...	0156	17·4	04·36	45·13
Published values	...	0179	19·4	04·51	45·33
1937 Adjustment	...	0135	17·4	04·28	45·22

* See Appendix III, para 6.

(Contd.)

TABLE 1 A—(*Contd.*)

India

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
	Station 10.	*Khārko–Gandpahār*			
Sironj terms	...	4·802 3964	12 11 57·4	27 35 15·20	67 33 12·76
Adjustment A	...	3964	55·4	15·24	12·53
Published values	...	4023	57·2	15·41	12·78
1937 Adjustment	...	4004	55·4	15·16	12·62
	Station 11.	*Zāwa–Zibra*			
Sironj terms	...	5·031 8600	178 40 52·4	28 57 44·20	66 35 18·74
Adjustment A	...	8585	50·7	44·20	18·47
Published values	...	8604	51·9	44·43	18·70
1937 Adjustment	...	8625	50·7	44·17	18·53
	Station 12.	*Pulchotau–Kisanen Chappar*			
Sironj terms	R.H.	5·370 9001	79 27 33·7	29 11 06·64	65 05 47·81
	L.H.	8945	32·0	06·66	47·73
Adjustment A	R.H.	8983	32·2	06·61	47·55
	L.H.	8983	32·2	06·71	47·44
Published values	...	9007	33·3	06·86	47·77
1937 Adjustment	...	8983	32·2	06·58	47·58
	Station 13.	*Tozghi–Shuri*			
Sironj terms	...	5·124 2565	216 16 39·6	28 53 14·18	62 14 59·02
Adjustment A	...	2547	39·4	14·12	58·81
Published values	...	2571	39·2	14·38	58·96
1937 Adjustment*	...	2547	39·4	14·09	58·84
	Station 14.	*Kūh-i-Malik Siāh–Kācha Kūh*			
Sironj terms	...	5·176 5550	321 52 02·6	29 51 31·75	60 52 19·78
Adjustment A	...	5532	02·4	31·67	19·58
Published values	...	5556	02·3	31·95	19·71
1937 Adjustment*	...	5532	02·4	31·64	19·62
	Station 15.	*Buzgalaband–Kapar*			
Sironj terms	...	5·082 6731	68 28 03·4	26 30 04·18	65 37 28·01
Adjustment A	...	6769	02·1	04·15	27·77
Published values	...	6793	03·4	04·33	27·93
1937 Adjustment	...	6749	02·1	04·07	27·89
	Station 16.	*Gundak–Basha*			
Sironj terms	...	4·869 5416	276 29 27·2	31 09 49·24	67 23 28·62
Adjustment A	...	5392	25·0	49·23	28·23
Published values	...	5403	26·9	49·49	28·55
1937 Adjustment	...	5472	25·0	49·30	28·34
	Station 17.	*Yūsuf–Husain Khān*			
Sironj terms	...	4·710 5965	320 31 09·9	27 51 08·50	68 26 14·70
Adjustment A	...	5955	08·1	08·56	14·46
Published values	...	6015	09·3	08·74	14·75
1937 Adjustment*	...	6053	08·1	08·50	14·60

* See Appendix III. para 6.

(*Contd.*)

TABLE 1 A—(*Contd.*)

India

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
	Station 18.	*Dāowāla–Māchka*			
Sironj terms	R.H.	4·778 0708	87 01 26·7	28 20 12·37	69 50 30·58
	L.H.	0759	28·1	12·59	30·60
Adjustment A	R.H.	0732	26·1	12·42	30·52
	L.H.	0732	26·1	12·69	30·32
Published values	...	0783	26·7	12·87	30·68
1937 Adjustment	...	0872	26·0	12·65	30·61
	Station 19.	*Garinda–Gūglā-Bhar*			
Sironj terms	...	4·853 6646	180 21 20·5	27 55 30·63	75 01 18·41
Adjustment A	...	6606	22·9	30·55	18·50
Published values	...	6621	21·6	30·55	18·47
1937 Adjustment	...	6628	22·9	30·55	18·44
	Station 20.	*Kunandwāla–Tamālāwāla*			
Sironj terms	...	4·742 8226	124 04 47·9	30 39 37·92	75 01 53·68
Adjustment A	...	8161	48·9	37·72	53·86
Published values	...	8272	50·4	37·85	53·83
1937 Adjustment	...	8201	48·9	37·80	53·75
	Station 21.	*Moni-Dhai–Akbar-da-Būnga*			
Sironj terms	...	4·804 4765	85 43 44·1	30 13 20·03	73 40 56·14
Adjustment A	...	4735	44·5	19·87	56·37
Published values	...	4839	45·9	19·98	56·21
1937 Adjustment	...	4815	43·5	19·90	56·18
	Station 22.	*Kanda–Kaimsir*			
Sironj terms	R.H.	4·802 1172	73 26 33·3	29 27 41·57	72 19 45·17
	L.H.	1158	30·5	41·38	44·87
Adjustment A	R.H.	1172	33·3	41·42	45·41
	L.H.	1172	33·3	41·42	45·00
Published values	...	1263	34·6	41·52	45·11
1937 Adjustment	...	1252	33·3	41·39	45·14
	Station 23.	*Lanjiwār–Chuharlār*			
Sironj terms	R.H.	4·744 2477	240 18 47·6	28 48 21·08	70 29 22·49
	L.H.	2578	50·8	20·70	22·20
Adjustment A	R.H.	2542	48·8	20·93	22·67
	L.H.	2542	48·8	20·80	21·88
Published values	...	2578	48·6	21·00	22·27
1937 Adjustment*	...	2642	47·8	20·82	22·23
	Station 24.	*Langāwāla–Tounsa*			
Sironj terms	R.H.	4·794 6347	21 09 54·8	30 51 26·90	70 43 18·05
	L.H.	6425	58·0	26·60	18·00
Adjustment A	R.H.	6301	55·3	26·84	18·28
	L.H.	6391	55·3	26·71	17·55
Published values	...	6400	54·8	26·93	17·89
1937 Adjustment	...	6411	53·8	26·82	17·80

* See Appendix III, para 6.

(*Contd.*)

TABLE 1 A—(*Contd.*)

India

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
	Station 25.	*Surla–Pathrijāla*			
Sironj terms	R.H.	5·135 2407	135 37 23·6	33 23 20·71	72 36 59·36
	L.H.	2412	26·6	20·72	59·65
Adjustment A	R.H.	2416	25·2	20·55	59·63
	L.H.	2416	25·2	20·72	59·90
Published values	...	2439	25·7	20·85	59·49
1937 Adjustment	...	2416	26·2	20·71	59·44
	Station 26.	*Rasia–Mānpur*			
Sironj terms	...	4·973 5997	219 49 46·2	27 26 16·98	77 10 24·78
Adjustment A	...	5967	44·5	16·93	24·74
Published values	...	5965	45·2	16·92	24·77
1937 Adjustment	...	5967	44·5	16·93	24·74
	Station 27.	*Banog–Amsot*			
Sironj terms	...	5·038 8076	71 06 12·9	30 28 37·05	78 00 56·10
Adjustment A	...	8019	09·5	36·93	55·86
Published values	...	8051	09·2	36·91	55·96
1937 Adjustment	...	8019	09·5	36·93	55·86
	Station 28.	*Choti Khanowri–Anetu*			
Sironj terms	...	4·730 3441	210 58 13·7	29 49 32·76	76 05 41·39
Adjustment A	...	3456	13·6	32·80	41·39
Published values	...	3490	14·8	32·88	41·40
1937 Adjustment	...	3466	13·6	32·88	41·36
	Station 29.	*Barāol–Bārādevi*			
Sironj terms	R.H.	5·157 8935	248 48 47·9	31 03 04·77	76 27 26·39
	L.H.	8843	41·2	04·37	26·19
Adjustment A	R.H.	8863	42·2	04·51	26·18
	L.H.	8863	42·2	04·43	26·20
Published values	...	8901	44·0	04·54	26·28
1937 Adjustment	...	8863	42·2	04·51	26·18
	Station 30.	*Gurhāgarh–Sāmnābanj*			
Sironj terms	R.H.	5·091 6316	253 53 03·5	32 37 59·62	75 02 05·22
	L.H.	6441	02·0	38 00·01	05·01
Adjustment A	R.H.	6318	05·8	37 59·72	05·26
	L.H.	6318	05·8	59·66	05·30
Published values	...	6379	07·0	59·94	05·30
1937 Adjustment	...	6358	06·8	59·83	05·22
	Station 31.	*Jogi Tila–Kandi*			
Sironj terms	R.H.	5·331 6243	220 08 57·7	32 51 33·50	73 26 23·05
	L.H.	6092	09 01·9	33·46	23·33
Adjustment A	R.H.	6108	02·5	33·22	23·52
	L.H.	6108	02·5	33·28	23·59
Published values	...	6132	03·7	33·56	23·48
1937 Adjustment*	...	6128	03·5	33·43	23·41
	Station 32.	*Ismāil di Dōri–Kakwa-ka-Pahār*			
Sironj terms	...	4·812 2536	338 50 02·2	34 29 44·07	73 55 15·93
Adjustment A	...	2552	02·8	43·91	16·22
Published values	...	2576	04·0	44·21	16·16
1937 Adjustment	...	2572	03·8	44·07	16·08

* See Appendix III, para 6.

(*Contd.*)

TABLE 1 A—(*Contd.*)

INDIA

Scale, Azimuth, Latitude and Longitude at junction points

Station 33. Gāshu Shish–Chamūri		° ′ ″	° ′ ″	° ′ ″
Sironj terms ...	4·987 7130	306 56 12·5	35 44 03·97	74 16 31·14
Adjustment A ...	7146	13·1	03·83	31·46
Published values ...	7170	14·3	04·14	31·43
1937 Adjustment* ...	7166	14·1	04·01	31·35
Station 34. Sar-bulak–Kok-tek				
Sironj terms ...	4·369 2822	68 13 10·3	37 18 58·72	75 04 40·79
Adjustment A ...	2836	10·9	58·59	41·14
Published values ...	3768	00·4	59·15	41·29
1937 Adjustment *† ...	2858	11·9	58·79	41·08
Station 35. Kalūmar–Lora				
Sironj terms ...	5·155 8732	265 29 55·3	23 27 52·28	79 44 23·82
Adjustment A ...	8722	54·6	52·30	23·82
Published values ...	8720	55·7	52·28	23·80
1937 Adjustment ...	8722	54·6	52·30	23·81
Station 36. Chamki–Mūrergarh				
Sironj terms ...	5·038 8884	265 26 50·5	23 34 09·97	81 38 16·14
Adjustment A ...	8864	49·5	10·01	16·11
Published values ...	8877	50·7	09·96	16·11
1937 Adjustment ...	8864	51·0	09·99	16·10
Station 37. Bhursu–Hariharpur				
Sironj terms ...	4·991 5664	267 56 34·1	23 15 57·14	84 44 19·29
Adjustment A ...	5634	32·8	57·24	19·21
Published values ...	5640	34·0	57·13	19·21
1937 Adjustment ...	5614	33·8	57·16	19·15
Station 38. Tilabani–Sūsinia				
Sironj terms ...	5·162 4598	272 58 27·3	23 24 59·87	86 33 14·74
Adjustment A ...	4558	25·7	25 00·01	14·60
Published values ...	4569	27·0	24 59·87	14·64
1937 Adjustment ...	4528	25·7	59·91	14·51
Station 39. Calcutta Base South–Calcutta Base North				
Sironj terms ...	4·530 9712	177 10 36·9	22 36 55·65	88 22 54·58
Adjustment A ...	9654	30·3	55·90	54·41
Published values ...	9666	36·2	55·68	54·43
1937 Adjustment ...	9654	30·3	55·82	54·30
Station 40. Harnkuli–Patna				
Sironj terms ...	4·747 2740	136 26 28·6	21 40 39·60	87 18 34·27
Adjustment A ...	2690	22·0	39·78	34·26
Published values ...	2707	26·8	39 64	34·17
1937 Adjustment* ...	2710	23·0	39·70	34·14
Station 41. Daiteri–Baniājori				
Sironj terms ...	5·165 5696	216 22 30·4	21 06 23·26	85 48 32·65
Adjustment A ...	5651	23·7	23·31	32·67
Published values ...	5672	26·1	23·24	32·52
1937 Adjustment ...	5651	25·7	23·25	32·52

* See Appendix III, para 6.

† These figures (1937) ignore the Russian base-line at Kizil Rabāt.

(*Contd.*)

TABLE 1 A—(*Contd.*)

INDIA

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
	Station 42.	*Himāgiri–Deodongar*			
Sironj terms	R.H.	4·999 3015	252 00 17·2	18 49 27·39	83 47 06·70
	L.H.	3047	20·1	27·46	06·44
Adjustment A	R.H.	3007	13·2	27·51	06·85
	L.H.	3007	13·2	27·36	06·89
Published values	...	3020	13·9	27·29	06·69
1937 Adjustment	...	3017	13·9	27·39	06·74
	Station 43.	*Gumru–Mārki*			
Sironj terms	R.H.	4·849 6330	126 07 08·7	17 56 05·97	83 14 07·26
	L.H.	6332	09·7	06·05	07·35
Adjustment A	R.H.	6325	04·5	06·15	07·44
	L.H.	6325	04·5	06·14	07·58
Published values	...	6341	05·6	05·91	07·39
1937 Adjustment	...	6327	04·5	06·01	07·45
	Station 44.	*Jogijogan–Mouwa*			
Sironj terms	...	5·159 4741	97 46 47·3	22 11 34·83	84 07 48·16
Adjustment A	...	4721	45·1	34·93	48·11
Published values	...	4748	47·4	34·81	48·08
1937 Adjustment*	...	4721	46·0	34·87	48·05
	Station 45†.	*Singhijuba–Chirguni*			
Sironj terms	R.H.	5·129 5840	272 34 19·6	21 03 32·64	83 45 09·65
Adjustment A	R.H.	5824	16·7	32·76	09·67
Published values	...	5868	19·6	32·61	09·57
1937 Adjustment	...	5864	17·4	32·67	09·59
	Station 45†.	*Chandli–Raun*			
Sironj terms	L.H.	5·007 4609	281 44 31·1	21 00 48·89	83 55 59·84
Adjustment A	L.H.	4564	24·4	48·76	56 00·04
Published values	...	4585	26·8	48·75	55 59·85
1937 Adjustment	...	4649	27·4	48·77	59·82
	Station 46.	*Somtāna–Shivālingāpa*			
Sironj terms	...	5·089 8495	8 59 26·1	19 05 00·47	77 39 16·22
Adjustment A	..	8505	24·5	00·45	16·29
Published values	...	8493	25·3	00·52	16·29
1937 Adjustment	...	8495	24·0	00·52	16·35
	Station 47.	*Dāmargīda–Dūdāla*			
Sironj terms	...	4·925 4110	300 41 24·1	18 03 17·29	77 40 04·33
Adjustment A	...	4132	22·1	17·26	04·44
Published values	...	4126	23·7	17·35	04·41
1937 Adjustment	...	4132	22·1	17·33	04·49
	Station 48.	*Rāmgīr–Timāpuram*			
Sironj terms	R.H.	5·153 5760	161 35 47·2	18 35 25·98	79 31 42·38
	L.H.	5740	44·7	26·03	42·40
Adjustment A	R.H.	5710	44·2	26·03	42·44
	L.H.	5710	44·2	26·13	42·46
Published values	...	5711	44·9	26·12	42·36
1937 Adjustment	...	5710	44·3	26·16	42·44

* See Appendix III, para 6.

† Singhijuba and Chirguni are stations of the Sambalpur meridional series. Chandli and Raun are stations of the Sambalpur longitudinal series. There is no complete connection between the two series, but Chandli and Raun were observed as intersected points of the meridional series, and so provide a satisfactory connection in latitude and longitude, but not in scale or azimuth.

(*Contd.*)

TABLE 1 A—(*Contd.*)

India

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
	Station 49.	*Pānch Pandol–Katājpur*			
Sironj terms	R.H.	5·281 9415	128 01 49·6	17 47 16·02	80 15 19·43
	L.H.	9486	49·4	15·97	19·65
Adjustment A	R.H.	9442	48·9	16·04	19·59
	L.H.	9442	48·9	16·10	19 69
Published values	...	9459	49·5	16·07	19·59
1937 Adjustment	...	9462	47·9	16·13	19·69
	Station 50.	*Cheru–Pāncha*			
Sironj terms	R.H.	5·024 2320	133 09 08·6	17 52 22·77	82 08 51·24
	L.H.	2288	13·6	22·99	51·04
Adjustment A	R.H.	2284	10·1	22·87	51·22
	L.H.	2284	10·1	23·10	51·23
Published values	...	2312	10·0	22·84	51·16
1937 Adjustment	...	2304	09·1	22·95	51·24
	Station 51.	*Singi–Pārner*			
Sironj terms	R.H.	5·414 3986	262 05 16·2	18 56 45·36	73 39 43·24
	L.H.	3942	22·8	45·78	43·02
Adjustment A	R.H.	3939	20·8	45·47	43·02
	L.H.	3939	20·8	45·58	43·01
Published values	...	3940	23·8	45·89	43·12
1937 Adjustment	...	3939	20·8	45·66	43·06
	Station 52.	*Khānpisura–Alsunda*			
Sironj terms	...	5·139 6332	324 49 52·7	18 45 30·55	74 46 49·72
Adjustment A	...	6362	49·5	30·41	49·73
Published values	...	6322	54·0	30·65	49·81
1937 Adjustment	...	6362	49·5	30·48	49·79
	Station 53.	*Kudurĕmukha–Ammĕdikal*			
Sironj terms	...	5·015 4395	301 55 51·1	13 07 40·20	75 15 56·19
Adjustment A	...	4429	49·2	39·93	56·47
Published values	...	4410	53·1	40·32	56·08
1937 Adjustment	...	4409	49·2	40·04	56·45
	Station 54.	*Savandurga–Rāmadevarabĕtta*			
Sironj terms	R.H.	5·185 8412	162 48 34·5	12 55 05·87	77 17 35·89
	L.H.	8442	38·1	05·94	35·60
Adjustment A	R.H.	8445	31·1	05·72	36·24
	L.H.	8445	31·1	05·82	36·09
Published values	...	8445	36·4	05·92	35·81
1937 Adjustment	...	8445	31·1	05·83	36·21
	Station 55.	*Māvandūr–Kurumkota*			
Sironj terms	R.H.	5·050 3691	194 50 55·1	12 44 37·40	79 39 59·10
	L.H.	3687	50·5	37·52	59·31
Adjustment A	R.H.	3694	47·7	37·57	59·62
	L.H.	3694	47·7	37·46	59·60
Published values	...	3713	52·2	37·47	59·35
1937 Adjustment	...	3694	47·7	37·58	59·73
	Station 56.	*Māniam–Dhūlipalla*			
Sironj terms	...	4·882 8301	253 24 11·7	16 22 20·84	79 52 57·59
Adjustment A	...	8321	10·7	20·83	57·77
Published values	...	8336	11·6	20·85	57·70
1937 Adjustment	...	8311	09·7	20·92	57·89

(*Contd.*)

TABLE 1 A—(*Contd.*)

India

Scale, Azimuth, Latitude and Longitude at junction points

	Station 57.	*Dhār–Sānjib*			
			° ′ ″	° ′ ″	° ′ ″
Sironj terms	R.H.	5·030 7296	315 34 21·9	17 43 59·02	82 28 27·63
	L.H.	7311	25·5	59·21	27·68
Adjustment A	R.H.	7307	22·0	59·04	27·83
	L.H.	7307	22·0	59·34	27·88
Published values	...	7325	22·1	59·10	27·81
1937 Adjustment*	...	7327	21·0	59·20	27·90
	Station 58.	*Karadigutta–Guthirāyan*			
Sironj terms	...	4·994 2177	175 37 53·9	11 59 28·55	77 52 36·37
Adjustment A	...	2180	46·4	28·51	36·98
Published values	...	2173	53·0	28·54	36·60
1937 Adjustment	...	2160	47·4	28·52	37·10
	Station 59.	*Kalugumalai–Mudimalai*			
Sironj terms	...	5·026 6135	215 33 19·6	10 10 07·94	78 00 49·05
Adjustment A	...	6138	11·5	07·91	49·90
Published values	...	6127	19·3	07·93	49·29
1937 Adjustment	...	6108	12·5	07·96	49·98
	Station 60.	*Kutaitatti–Koilpati*			
Sironj terms	R.H.	5·162 6752	220 24 55·5	8 51 21·95	77 35 51·91
	L.H.	6773	25 04·5	22·23	51·49
Adjustment A	R.H.	6754	24 47·1	21·85	52·94
	L.H.	6754	47·1	21·99	52·82
Published values	...	6746	55·2	21·96	52·17
1937 Adjustment	...	6754	47·1	21·92	53·00
	Station 61.	*Kudankulam–Rādhāpuram*			
Sironj terms	...	4·605 4754	185 55 26·2	8 10 21·54	77 41 26·00
Adjustment A	...	4756	18·7	21·44	27·13
Published values	...	4748	25·9	21·55	26·26
1937 Adjustment*	...	4756	18·7	21·51	27·19
	Station 62.	*Mallipat–Kiliyūr*			
Sironj terms	R.H.	4·885 2305	36 56 23·0	11 58 00·31	79 22 33·55
	L.H.	2310	29·5	00·23	33·55
Adjustment A	R.H.	2300	21·0	00·39	34·16
	L.H.	2300	21·0	00·37	34·18
Published values	...	2339	25·8	00·26	33·84
1937 Adjustment	...	2300	21·0	00·38	34·29
	Station 63.	*Pallathivayal–Kulamangalam*			
Sironj terms	R.H.	4·663 9501	188 21 09·4	10 09 11·25	79 01 00·78
	L.H.	9686	21 13·3	11·29	00·63
Adjustment A	R.H.	9636	20 57·8	11·38	01·69
	L.H.	9636	20 57·8	11·42	01·56
Published values	...	9698	21 05·9	11·23	01·10
1937 Adjustment	...	9656	20 57·8	11·40	01·75
	Station 64.	*Rāmnād–Sambuttiyendal*			
Sironj terms	...	4·643 6408	101 00 59·8	9 21 52·07	78 49 17·08
Adjustment A	...	6374	43·2	52·17	18·27
Published values	...	6396	51·1	51·96	17·66
1937 Adjustment	...	6424	43·2	52·12	18·45

* See Appendix III, para 6.

(*Contd.*)

TABLE 1 A—(*Concld.*)

India

Scale, Azimuth, Latitude and Longitude at junction points

Station 65. Kachi Tīvu North–Pisāsu Mundal				
		° ′ ″	° ′ ″	° ′ ″
Sironj terms ...	4·904 2195	72 52 24·7	9 23 29·94	79 31 28·46
Adjustment A ...	2161	08·1	30·24	29·62
Published values ...	2183	16·0	29·94	29·03
1937 Adjustment* ...	2211	08·1	30·20	29·83

* See Appendix III, para 6.

TABLE 1B

ASSAM–BURMA (*Provisional*)

Scale, Azimuth, Latitude and Longitude at junction points

	Station 66. Chinsura–Sātten				
			° ′ ″	° ′ ″	° ′ ″
Sironj terms	...	4·809 7537	122 03 48·0	22 52 55·85	88 24 11·45
Adjustment A	...	7479	41·4	56·09	11·25
Published values	...	7497	47·4	55·87	11·33
1937 Adjustment*	...	7479	41·4	56·01	11·14
	Station 67. Orfi–Hatiāra				
Sironj terms	...	4·751 3276	205 51 52·8	23 01 06·99	89 47 56·06
Adjustment A	...	3296	54·6	07·28	55·83
Published values	...	3310	54·4	06·98	55·97
1937 Adjustment	...	3246	54·6	07·17	55·70
	Station 68. Lakhinagar–Bāshakpur				
Sironj terms	...	4·722 8172	269 05 14·6	23 00 39·78	90 45 43·16
Adjustment A	...	8192	20·3	40·05	42·95
Published values	...	8198	19·8	39·73	43·08
1937 Adjustment*	...	8237	20·3	39·94	42·82
	Station 69. Gojalia–Sāhebmura				
Sironj terms	...	4·917 9281	199 24 51·9	23 09 04·88	91 33 31·49
Adjustment A	...	9301	55·9	05·08	31·31
Published values	...	9331	59·1	04·76	31·47
1937 Adjustment	...	9431	55·9	04·99	31·24
	Station 70. Kailās Tīla–Bar Utni Tīla				
Sironj terms	...	4·802 4449	168 51 06·6	24 47 41·28	92 01 46·42
Adjustment A	...	4469	12·6	41·47	46·39
Published values	...	4460	18·3	41·13	46·70
1937 Adjustment	...	4469	12·6	41·46	46·34
	Station 71. Newāni–Rāmganj				
Sironj terms	...	4·821 0298	105 28 31·6	26 16 00·74	88 29 12·00
Adjustment A	...	0216	28·6	00·78	11·48
Published values	...	0218	33·5	00·95	11·92
1937 Adjustment	...	0216	25·6	00·89	11·36
	Station 72. Dhubri–Alangjāni				
Sironj terms	R.H.	4·894 5604	81 23 26·6	26 01 03·74	89 59 51·38
	L.H.	5834	24·2	03·61	51·52
Adjustment A	R.H.	5729	22·6	04·21	51·10
	L.H.	5729	22·6	03·73	50·90
Published values	...	5653	25·6	03·82	51·26
1937 Adjustment	...	5629	17·6	03·94	50·75
	Station 73. Tepkilabama–Harogaon				
Sironj terms	R.H.	4·675 7308	89 58 42·4	25 56 22·33	91 34 25·16
	Centre	7289	38·5	22·26	25·23
	L.H.	7187	51·6	22·09	25·85
Adjustment A	R.H.	7301	47·0	22·54	25·27
	Centre	7301	47·0	22·52	25·34
	L.H.	7301	47·0	22·64	25·75
Published values	...	7245	51·2	22·19	25·79
1937 Adjustment	...	7251	47·0	22·49	25·30

* See Appendix III, para 6.

(*Contd.*)

TABLE 1 B—(*Contd.*)

Assam–Burma (*Provisional*)

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
Station 74. Golāghāt–Cheniābinshon					
Sironj terms	...	4·946 6600	59 21 27·2	26 30 48·16	93 57 27·23
Adjustment A	...	6590	30·9	48·21	27·38
Published values	...	6623	44·2	47·64	27·88
1937 Adjustment	...	6540	30·9	48·14	27·32
Station 75. Nāginimāra–Lirumen					
Sironj terms	...	5·075 3219	59 57 37·9	26 48 58·00	94 50 00·25
Adjustment A	...	3207	41·1	58·00	00·42
Published values †	...	...	...	...	...
1937 Adjustment	...	3207	41·2	57·93	00·34
Station 76. Lungwuka Bum–Nāseng Bum					
Sironj terms	...	5·023 2863	250 47 13·4	26 29 21·78	95 50 57·39
Adjustment A	...	2852	16·1	21·73	57·55
Published values †	...	...	...	...	...
1937 Adjustment	...	2792	16·1	21·66	57·44
Station 77. Merpa Tīla–Dali Tīla					
Sironj terms	...	4·797 5301	354 50 44·7	25 01 45·44	92 20 40·11
Adjustment A	...	5321	50·4	45·61	40·12
Published values	...	5312	56·3	45·23	40·45
1937 Adjustment*	...	5316	50·4	45·60	40·07
Station 78. Tukbai–Rāmphan					
Sironj terms	...	4·900 5324	12 59 33·7	25 00 56·73	93 06 38·62
Adjustment A	...	5344	38·7	56·83	38·64
Published values	...	5292	44·8	56·38	38·95
1937 Adjustment*	...	5329	38·7	56·82	38·58
Station 79. Khambi Ching–Tamunja					
Sironj terms	...	4·969 8597	296 30 03·9	24 46 01·09	94 21 42·74
Adjustment A	...	8617	08·2	01·10	42·76
Published values	...	8512	14·2	00·54	42·93
1937 Adjustment	...	8587	08·2	01·10	42·68
Station 80. Sirohifara–Khambi Ching					
Sironj terms	...	5·105 2280	14 48 40·8	25 06 21·55	94 27 36·17
Adjustment A	...	2300	45·1	21·56	36·22
Published values	...	2189	51·3	20·95	36·43
1937 Adjustment*	...	2270	45·1	21·55	36·19
Station 81. Taungthonlon–Loi Maw					
Sironj terms	R.H.	5·195 2880	279 45 28·3	24 57 30·23	95 48 22·93
	L.H.	2896	29·8	30·18	22·99
Adjustment A	R.H.	2890	31·3	30·16	22·98
	L.H.	2890	31·3	30·14	23·07
Published values	...	2805	39·3	29·43	23·05
1937 Adjustment	...	2900	31·3	30·15	22·90
Station 82. Bumdaw Bum–Bumsai Bum					
Sironj terms	R.H	5·238 1242	310 14 50·0	25 45 49·44	96 08 57·38
	L.H.	1277	50·4	49·46	57·47
Adjustment A	R.H.	1267	52·4	40·36	57·44
	L.H.	1267	52·4	49·40	57·57
Published values	...	1168	15 01·0	48·52	57·66
1937 Adjustment*	...	1207	14 54·4	40·38	57·42

* See Appendix III, para 6.
† Not yet published.

(*Contd.*)

TABLE 1 B—(*Contd.*)

ASSAM–BURMA (*Provisional*)

Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
		Station 83. Katha–Ubyētaung			
Sironj terms	...	5·325 7841	27 31 07·4	24 11 54·27	96 15 15·42
Adjustment A	...	7846	10·4	54·17	15·43
Published values	...	7767	18·3	53·44	15·36
1937 Adjustment*	...	7846	10·4	54·17	15·34
		Station 84. Laka Pum–Tangte			
Sironj terms	...	5·077 9404	132 28 19·9	23 42 28·69	97 10 47·18
Adjustment A	...	9409	22·1	28·55	47·17
Published values	...	9330	30·7	27·72	46·95
1937 Adjustment	...	9369	23·1	28·54	47·05
		Station 85. Mara Bum–Singleng Pum			
Sironj terms	...	5·037 5620	219 14 04·9	25 18 48·45	97 30 00·01
Adjustment A	...	5635	07·2	48·31	00·07
Published values	...	5546	15·8	47·32	00·09
1937 Adjustment*	...	5665	10·2	48·29	00·01
		Station 86. Loi Wanwa–Loi Anglawng			
Sironj terms	R.H.	5·163 8924	188 19 50·8	21 42 31·49	99 18 18·49
	L.H.	8788	52·1	31·17	18·38
Adjustment A	R.H.	8777	51·2	31·25	18·02
	L.H.	8777	51·2	31·02	18·35
Published values	...	8714	20 02·7	29·96	17·61
1937 Adjustment	...	8777	19 51·2	31·07	18·08
		Station 87. Sītāpahār–Gilasari			
Sironj terms	...	4·830 0340	305 00 16·3	22 29 28·39	92 10 03·61
Adjustment A	...	0360	18·9	28·55	03·40
Published values	...	0369	26·5	28·17	03·50
1937 Adjustment*	...	0480	18·9	28·39	03·39
		Station 88. Webula–Wone-lone-taung			
Sironj terms	R.H.	5·120 7898	3 38 08·9	23 01 48·31	93 55 49·64
	L.H.	7869	07·6	48·31	49·89
Adjustment A	R.H.	7914	12·6	48·39	49·49
	L.H.	7914	12·6	48·29	49·75
Published values	...	7807	17·4	47·94	49·79
1937 Adjustment	...	7949	12·6	48·30	49·61
		Station 89. Hlaingma Taung–Pya Nattaung			
Sironj terms	...	4·964 4999	25 34 40·4	22 21 03·17	94 15 51·65
Adjustment A	...	5039	45·4	03·10	51·46
Published values	...	4946	49·8	02·78	51·42
1937 Adjustment*	...	5009	42·4	03·12	51·35
		Station 90. Sheinmaga–Mingun			
Sironj terms	R.H.	4·916 1717	354 23 32·8	22 16 34·46	95 58 15·97
	L.H.	1906	29·2	34·58	16·22
Adjustment A	R.H.	1866	33·4	34·31	15·91
	L.H.	1866	33·4	34·53	16·11
Published values	...	1875	39·2	33·89	15·79
1937 Adjustment	...	1966	33·4	34·44	16·01

* See Appendix III, para 6.

(*Contd.*)

TABLE 1 B—(*Concld.*)

Assam–Burma (*Provisional*)

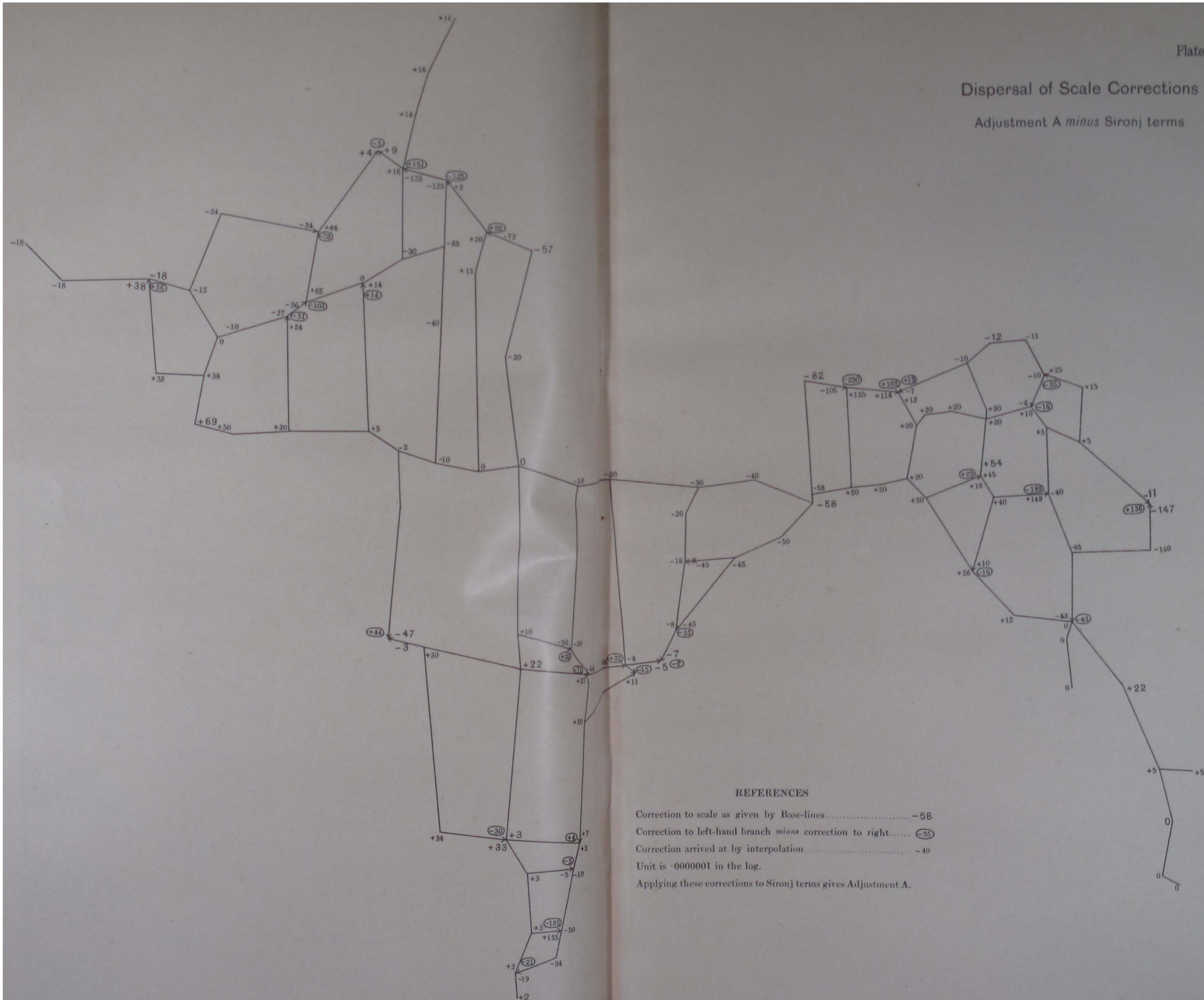
Scale, Azimuth, Latitude and Longitude at junction points

			° ′ ″	° ′ ″	° ′ ″
	Station 91. Rintaung–Retkamauk				
Sironj terms	R.H.	5·139 6154	345 44 11·0	20 09 44·12	93 22 18·14
	L.H.	6170	09·9	43·77	18·13
Adjustment A	R.H.	6180	16·7	44·15	17·78
	L.H.	6180	16·7	43·74	17·70
Published values	...	6136	21·7	43·64	17·51
1937 Adjustment	...	6204	16·7	43·83	17·78
	Station 92. Taungo–Kogwin Taung				
Sironj terms	...	4·917 9210	42 04 57·6	18 45 48·38	94 38 00·82
Adjustment A	...	9222	05 00·6	48·30	00·37
Published values	...	9147	10·1	47·67	37 59·77
1937 Adjustment	...	9222	03·6	47·94	38 00·35
	Station 93. Myayabengkyo–Thayetkyo				
Sironj terms	R.H.	5·237 0686	271 57 46·6	18 21 34·90	96 22 54·71
	L.H.	0729	43·3	34·47	54·68
Adjustment A	R.H.	0686	47·1	34·77	54·26
	L.H.	0686	47·1	34·57	54·32
Published values	...	0634	54·9	33·93	53·46
1937 Adjustment	...	0686	49·1	34·33	54·22
	Station 94. Sintaung–Lethataung				
Sironj terms	...	5·134 7248	337 27 48·7	20 31 17·82	96 31 29·17
Adjustment A	...	7183	51·7	17·81	28·92
Published values	...	7181	59·9	17·11	28·38
1937 Adjustment	...	7243	51·7	17·60	28·86
	Station 95. Loi Pakulin–Loi Tum				
Sironj terms	...	5·185 9970	263 06 22·0	20 20 38·27	99 00 27·06
Adjustment A	...	9830	23·0	38·19	26·60
Published values	...	9699	35·1	37·08	25·94
1937 Adjustment	...	9890	23·0	37·98	26·66
	Station 96. Sanwingan Taung–Kanyindaung				
Sironj terms	...	5·133 4517	246 28 58·7	17 54 28·29	96 02 50·86
Adjustment A	...	4517	59·2	28·16	50·41
Published values	...	4448	29 07·3	27·39	49·56
1937 Adjustment*	...	4517	01·2	27·73	50·36
	Station 97. Talokkōn–Syriam				
Sironj terms	...	5·054 7461	220 44 34·7	16 29 30·68	96 04 02·60
Adjustment A	...	7461	35·2	30·55	02·14
Published values	...	7392	43·3	29·86	01·08
1937 Adjustment*	...	7461	37·2	30·12	02·04
	Station 98. Taungzun–Kwanhla				
Sironj terms	...	4·903 7646	31 16 24·5	16 25 49·56	97 40 20·17
Adjustment A	...	7668	22·5	49·43	19·77
Published values	...	7593	32·9	48·48	18·54
1937 Adjustment	...	7608	22·5	48·96	19·70

* See Appendix III, para 6.

Dispersal of Scale Corrections

Adjustment A *minus* Sironj terms



REFERENCES

Correction to scale as given by Base-lines........................ −58

Correction to left-hand branch *minus* correction to right...... (−35)

Correction arrived at by interpolation............................ −40

Unit is ·0000001 in the log.

Applying these corrections to Sironj terms gives Adjustment A.

Scale 1 Inch = 225 Miles

Miles 100 0 100 200 300 400 Miles

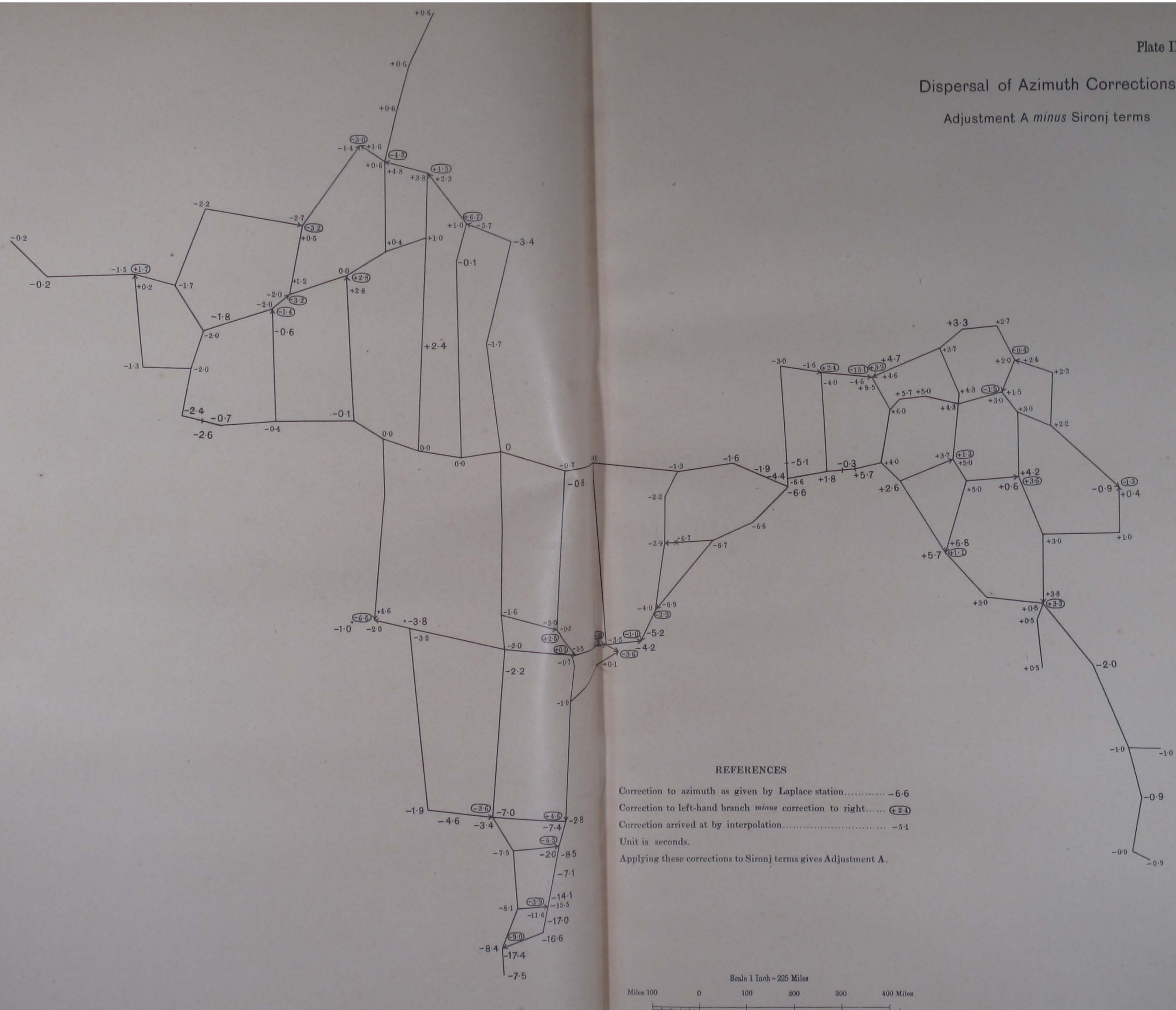
REG. No. 390D.D.D. 1938–350

Helio S. I. O. Dehra Dun.

Plate III

Dispersal of Azimuth Corrections

Adjustment A *minus* Sironj terms



REFERENCES

Correction to azimuth as given by Laplace station............ −6·6

Correction to left-hand branch *minus* correction to right...... (+2·4)

Correction arrived at by interpolation.............................. −5·1

Unit is seconds.

Applying these corrections to Sironj terms gives Adjustment A.

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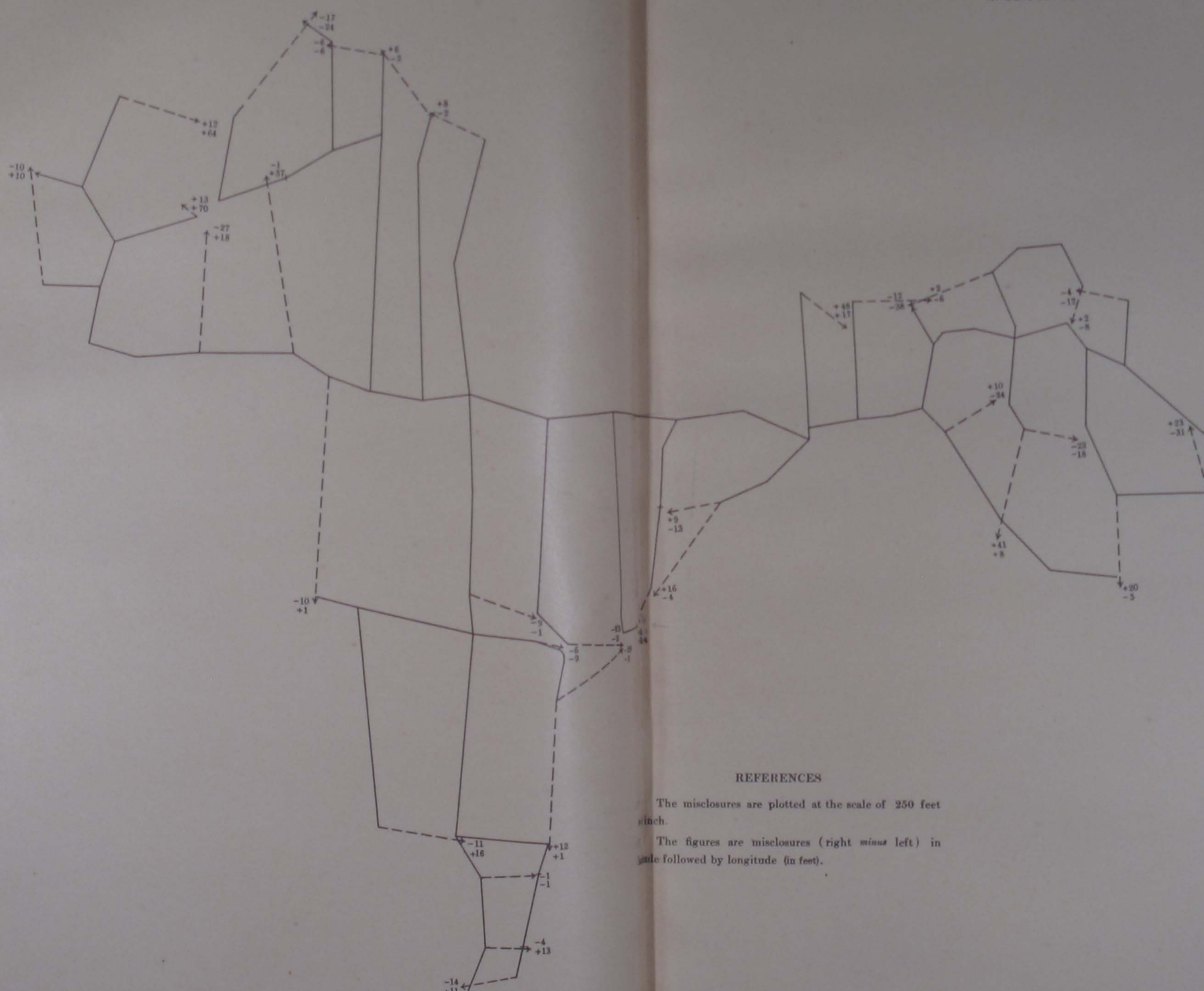
Adjustment A

Misclosures in latitude and longitude



REFERENCES

The misclosures are plotted at the scale of 250 feet inch.

The figures are misclosures (right *minus* left) in latitude followed by longitude (in feet).

Scale 1 Inch = 225 Miles

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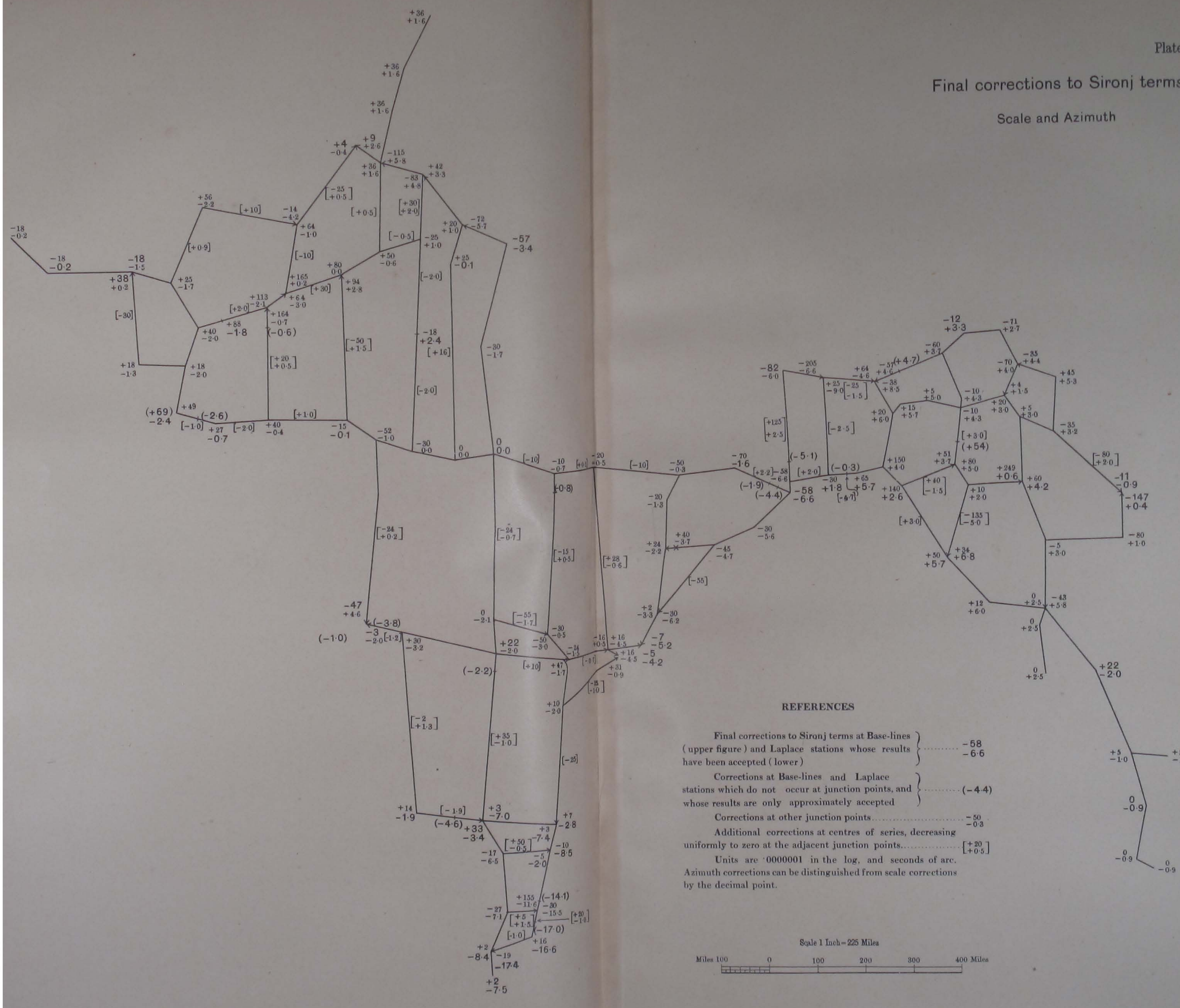
Plate V

Final corrections to Sironj terms

Scale and Azimuth



REFERENCES

Final corrections to Sironj terms at Base-lines (upper figure) and Laplace stations whose results have been accepted (lower) .... −58 / −6·6

Corrections at Base-lines and Laplace stations which do not occur at junction points, and whose results are only approximately accepted .... (−4·4)

Corrections at other junction points .... −50 / −0·3

Additional corrections at centres of series, decreasing uniformly to zero at the adjacent junction points .... [+20 / +0·5]

Units are ·0000001 in the log, and seconds of arc. Azimuth corrections can be distinguished from scale corrections by the decimal point.

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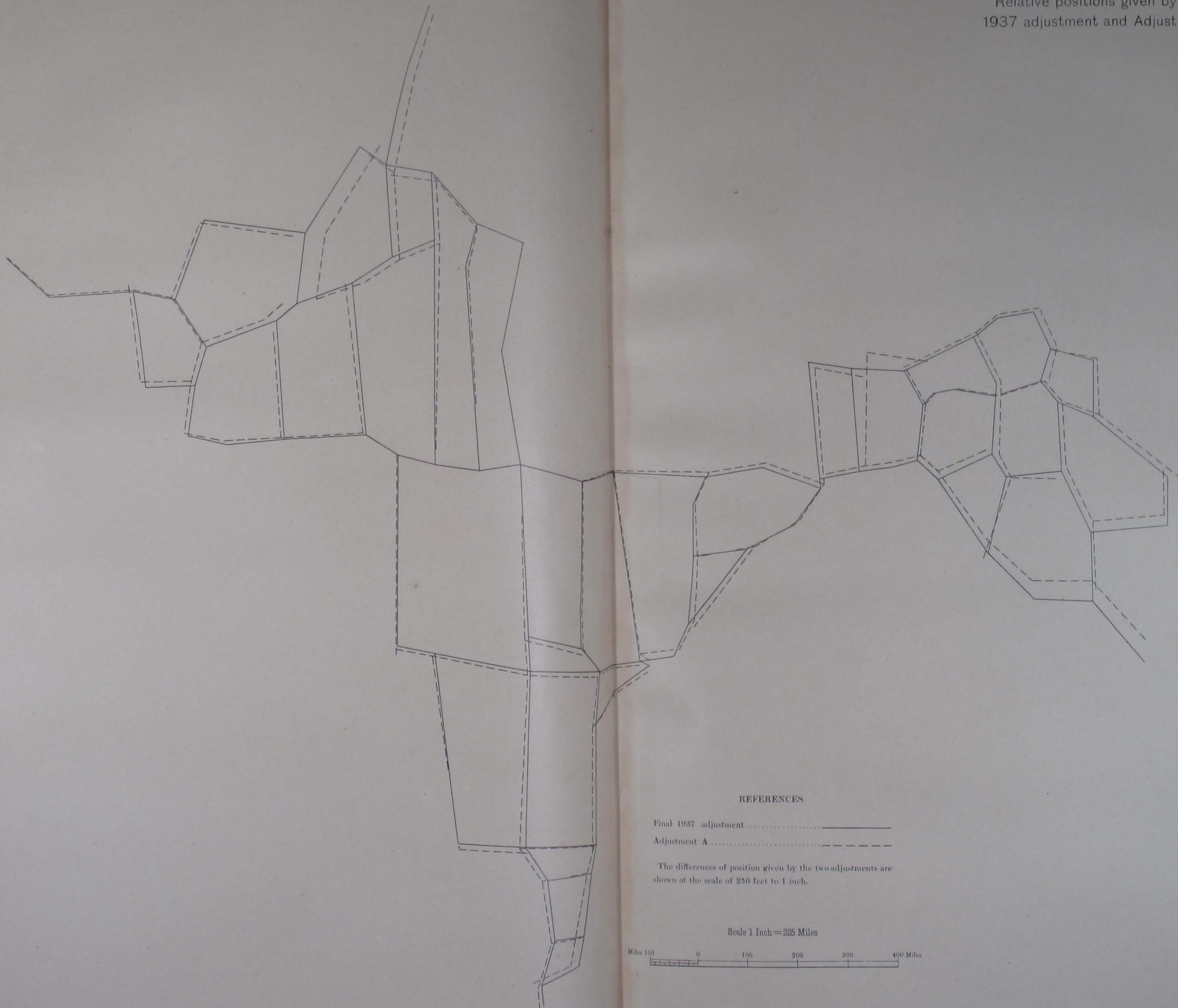
Relative positions given by Final 1937 adjustment and Adjustment A



REFERENCES

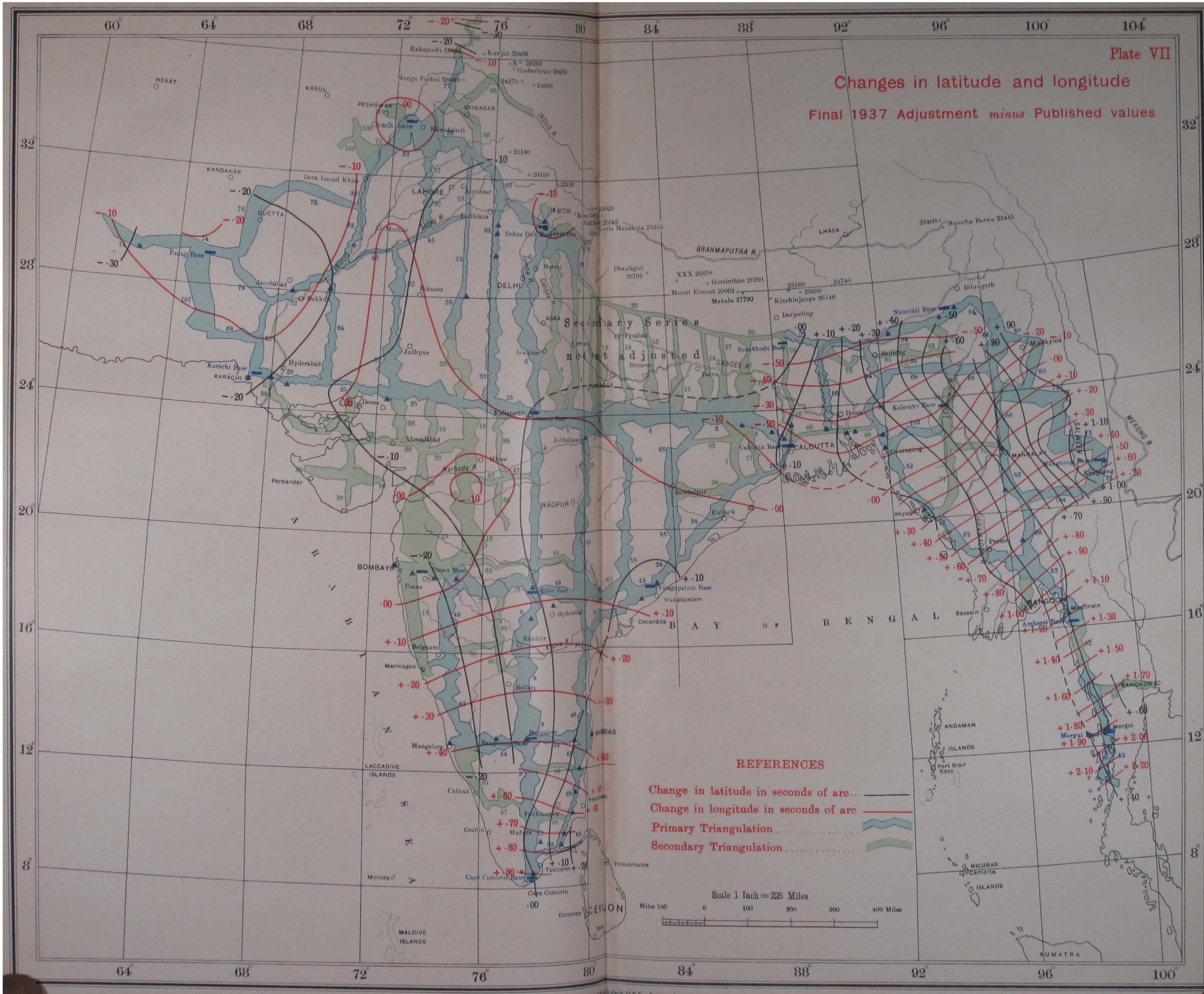
Final 1937 adjustment .... ————————

Adjustment A .... — — — — — —

The differences of position given by the two adjustments are shown at the scale of 250 feet to 1 inch.

Scale 1 Inch = 225 Miles

O.D. 1938-350.



REG. No. 398 D.D.D. 1938-350.

Published under the direction of Brigadier ... , O.B.E., Surveyor General of India

HELIO S.I.O. DEHRA DUN

SYMBOLS USED IN CHAPTER II, PARA 18, AND APPENDIX VII

e = probable error of an observed ($60°$) angle.

N_1 = probable error of scale after 100 miles, in units of 7th decimal of the log.

N_2 = probable error of azimuth after 100 miles, in seconds.

N = $\sqrt{N_1^2 + 443\, N_2^2}$. $443 = (4 \cdot 34 \times 10^6 \sin 1'')^2$.

s = probable error of base measurement and extension, in 7th decimal of the log.

t'' = probable error of a Laplace azimuth, in seconds.

u = $0 \cdot 75 s$.

v = $0 \cdot 75 t''$.

O = Origin from which errors are measured.

A = terminal point at which error is required.

B = an intermediate point.

Q = centre of gravity of the line representing the triangulation between O and A.

100 S = direct distance OA in miles.

100 R = distance from A to G, the middle point of any section, in miles.

100 ρ = distance from Q to G, the middle point of any section, in miles.

100 L = length of a straight section of the triangulation joining O and A, in miles.

θ = the angle between OA and a section of the triangulation joining O and A.

x = distance measured from A along AO.

$\triangle x$ = error of position in direction OA.

$\triangle y$ = error of position normal to OA.

E_1 = probable value of $\triangle x$.

E_2 = probable value of $\triangle y$.

E = $\sqrt{E_1^2 + E_2^2}$.

l = length of an element of a series, in miles.

a = number of such elements between O and A.

b = number of such elements between O and B, etc.

ϵ_m = error of measured ratio of the $(m+1)^{th}$ element to the m^{th}. (Ratio, not log).

η_m = error of measured angle between the $(m+1)^{th}$ element and the m^{th}. (Circular measure).

ϵ, η = probable values of ϵ_m and η_m in any series.

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Chapter II

PROBABLE ERRORS OF THE TRIANGULATION

12. **Object of the investigation.**—A knowledge of the liability of each triangulation series to accumulate error in scale, azimuth and position is necessary for two purposes :—

(*a*) In order that closing errors may be most reasonably distributed among the series contributing to them, as has been done in Chapter I.

(*b*) In order to assess the probable errors of the finally adjusted latitudes and longitudes, and hence to judge whether the triangulation is sufficiently accurate.

13. **Probable error of observed angles.**—It is first necessary to obtain a measure of the probable errors of the observed angles. In the 1880 adjustment a different weight was given to every angle of the triangulation, but for the present purpose that is thought to be unnecessary (and in any case impossible to do with any truth), and equal probable errors are assigned to all the angles of any piece of triangulation observed under uniform circumstances. This probable error can be obtained with increasing accuracy from four different considerations :—

(*a*) From the agreement of the different measures of each angle among themselves.

(*b*) From the triangular errors*.

(*c*) From the corrections given by the adjustment of figures more complex than simple triangles†.

(*d*) From the circuit, base and Laplace closures.

The last of these is the only method likely to give really reliable results, but it clearly cannot be used to determine the weights to be assigned during the adjustment of the circuit misclosures. The first method is obviously unreliable, since many sources of error are common to all the measures of a single angle, and the use of the triangular error is also liable to give results which are far too low. See Appendix VI, paras 4 (*e*) and (*h*). The figural adjustments are better, but often a series contains no figures other than simple triangles, and even in complex figures the triangular closures out-number the others.

The method now to be used is the third, from figural adjustments, except that when a series consists almost entirely of simple triangles the probable error is deduced by the second method from the triangular errors‡.

---

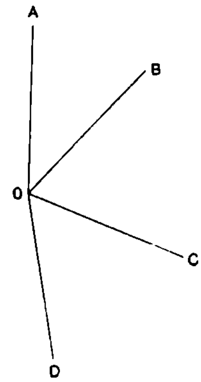
* P.E $= e = \cdot 674 m = \cdot 674 \sqrt{\Sigma \Delta^2 / 3n}$ where $\Sigma \Delta^2$ is the sum of the squares of the triangular errors, and n is the number of triangles. In G.T. Vol. II m is denoted by ϵ_2.

† For the method see G.T. Vol. II page 344. P.E $= e = \cdot 674 \epsilon_3 = \cdot 674 \sqrt{\Sigma w x^2 / w_0 n}$, where w is the weight assigned to each angle in the figural adjustment : x is the correction arising from the adjustment : n is the number of conditions : and w_0 is the mean value of all the w's considered. (Note. On page 344 $[wx]^2$ is a misprint for $[wx^2]$).

‡ In certain cases (*e.g.*, the NE. longitudinal series and the East Calcutta longitudinal series) neither of these methods appears to give a reliable result, and the probable error is assigned from other considerations. See Appendix VI, para 4.

The probable errors derived from these two methods are later multiplied by two augmenting factors deduced from the circuit closures. See para 19.

An insuperable obstacle to any accurate discussion of probable errors is the interdependence of all the angles observed at a station. If all angles were independently measured, precise discussion might be possible, but in India (as in most countries) angles are as far as possible measured in rounds, and the errors of adjacent angles are not independent*. Triangular errors and figural adjustments give the probable errors of angles (such as AOB in the marginal figure) whose average size is about 60°. It is important, especially from the point of view of azimuth (see para 15), to know the probable error of larger angles such as AOD. At first sight the probable errors of all the angles at a station, large or small, may be considered equal, but many classes of error tend to be larger in large angles than in small. From the point of view of internal evidence only, Appendix V concludes that the probable error of a 180° angle averages 1·25 that of a 60° angle, and this result is utilized in para 15, but para 19 suggests that the ratio is considerably larger.



14. Probable error of scale.—Formulæ are known† which give the probable error, after figural adjustment, of the scale and azimuth of the terminal side of any figure relative to the opening side, provided the probable error of each observed angle is known. In the general case, these formulæ are intolerably complicated, each figure involving the evaluation of a determinant with one more column than the figure has conditions, but simple formulæ apply in the case of simple regular figures whose angles are all of equal weight. The following results are deduced from G.T. Vol. II, p. 199:—

If e is the probable error of an observed angle in seconds, and if the length of the side of entry into the figure is errorless, the probable error of the log side of exit (expressed in units of ·000 0001) will be

For a pair of equilateral triangles ‡	...	24·8 e
For a regular braced quadrilateral	...	21·4 e
For a regular hexagon	...	27·6 e

Centered quadrilaterals may be treated as a pair of triangles, other single-centered figures as hexagons§, and double centered figures as 1½ hexagons. As noted in para 13, no great precision is possible: nor is it necessary.

The angles on which the continuity of scale mostly depends are generally small angles, of 60° or less, and the value of e obtained from the figural adjustments is appropriate for this purpose.

* In Geodetic Report Vol. VII page 147, Dr. J. de Graaff Hunter has drawn attention to one of the results of treating all measures as independent. On this assumption, the rigorous formula shows that some braced quadrilaterals would be strengthened by the omission of one diagonal, which is correct if angles are measured independently, but certainly not the case if angles are measured in rounds.

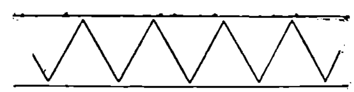
† J. de Graaff Hunter, Geodetic Report, Vol. VII, Chapter IX.

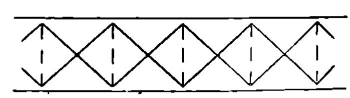
‡ For a single triangle G. T. Vol. II page 199 gives e.m.s. $= 0{\cdot}82\ R \sin 1'' \sqrt{u/\rho^2}$, where R = ratio of the 2 sides (= unity in this case), and $u/\rho^2 = (1/w)\ (\epsilon_3^2/\epsilon_1^2)$ page 344 $= \epsilon_1^2 \cdot \epsilon_3^2/\epsilon_1^2 = \epsilon_3^2$, where ϵ_3 is the finally accepted e.m.s. of an observed angle, and ϵ_1 is the preliminary value. Then p.e. of the ratio $= 0{\cdot}82\ e \sin 1'' = 17{\cdot}5\ e$ in terms of the 7th decimal of the log, and the p.e. after a pair of triangles is $\sqrt{2}$ times as much. It is convenient to treat a pair of triangles as a unit, rather than a single triangle.

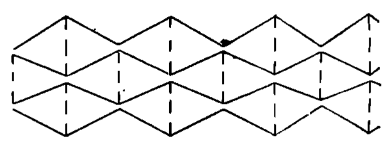
§ For centered quadrilaterals the correct figure is 24·6 e, for pentagons 25·8 e and for heptagons 30·2 e.

In the older series of the Indian triangulation, the figures have been laid out with great regularity, and the above formulæ can be applied to them with some accuracy, but during the last 30 years increased use has been made of more elongated figures, and some modification is necessary on this account. In Geodetic Report Vol. VII, page 147, Dr. J. de Graaff Hunter has given the rule that for an approximately rectangular figure, the inverse weight of the ratio (which varies as the square of its probable error) varies as l^2/b^2, where l is the length of the figure and b is its breadth. The probable error then varies as length ÷ breadth, and this convenient rule may reasonably be applied to all other figures, it being realized that it is not likely to be accurate in rare cases of extreme or unsymetrical elongation.

15. Probable error of azimuth.—As for scale, formulæ are available for the probable error of azimuth generated in any figure, but in view of the difficulty in estimating the probable error of the large angles on which continuity of azimuth largely depends, simpler treatment seems sufficient, if not actually more accurate.







The marginal figures represent series of simple triangles, quadrilaterals and hexagons respectively, and show that from the point of view of carrying forward azimuth these series of triangles can each be viewed as 3 or 4 traverses, whose angles are independently measured if the observations have been made on the usual system of rounds of angles.

In detail, the series of simple triangles consists of three traverses of which two, with 180° angles, contain one side per figure*, while the third with 60° angles contains two sides per figure. If the weight of an observed† 60° angle is unity, that of a 180° angle is $1/(1 \cdot 25)^2$ or $0 \cdot 64$, and the weight of the mean azimuth carried forward by a single figure is $0 \cdot 64 + 0 \cdot 64 + 0 \cdot 50 = 1 \cdot 78$. The probable error of the emergent azimuth is then $e/\sqrt{1 \cdot 78} = 0 \cdot 75\, e$.

Similarly, the series of quadrilaterals consists of four traverses, two with large angles and two with small, all with one side per figure, and the probable error after one figure is $0 \cdot 55\, e$.

For hexagons, there are four traverses, all with large angles (alternately 120° and 240°), and with two sides per figure. The probable error after one figure is then $0 \cdot 88\, e$, or $0 \cdot 62\, e$ per flank side, of which there are two per figure.

16. Comparison with M of Professional Paper No. 16.—In 1918 Dr. J. de Graaff Hunter introduced a quantity M‡ as a criterion of the accuracy of a series, and gave formulæ for the probable accumulation of error in scale and azimuth. From these formulæ can be deduced the probable errors after a single figure for comparison with those now proposed.

* Pair of triangles.

† The weight of an observed angle is required, not the weight of an angle after figural adjustment, for adjusted angles will no longer give independent "traverses". The figure 1·25 comes from Appendix V. It is probably too small, see para 19

‡ Survey of India Professional Paper No. 16, pages 91 and 105. $M = (1 + f)\, m \sqrt{18/l}$, where $m = \sqrt{\Sigma\Delta^2/3n}$, f is a fraction varying between zero and 1·6 according to the type of figure, and l miles is the average length of side in the series.

Formulæ for Probable Errors of Position

Fig. 1

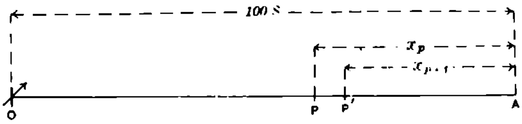


Fig. 3

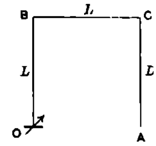


Fig. 2

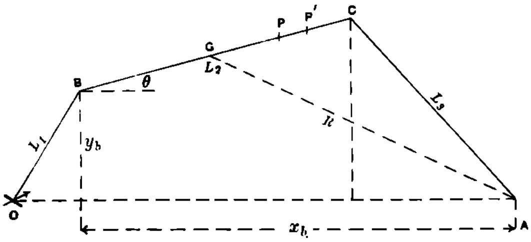


Fig. 4

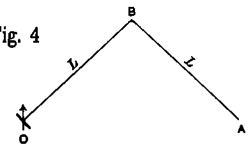


Fig. 5

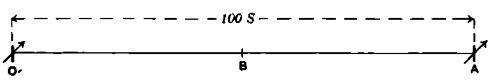


Fig. 6

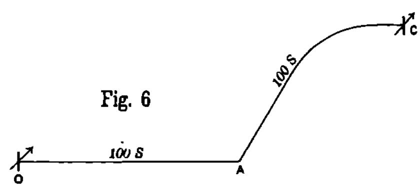


Fig. 7

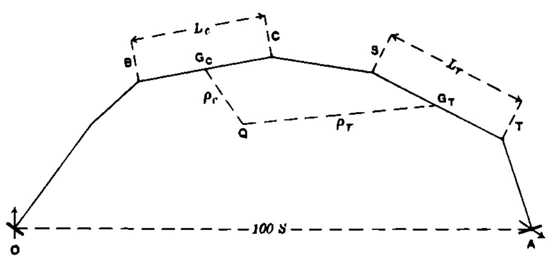


Fig. 8

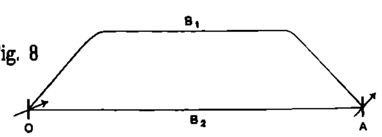


Fig. 9

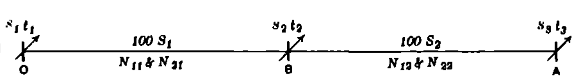


Fig. 11

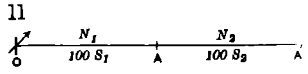


Fig. 10

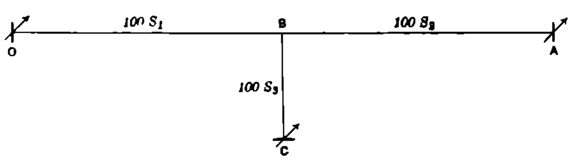


Fig. 12

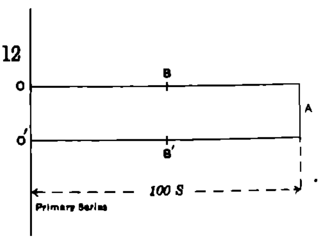
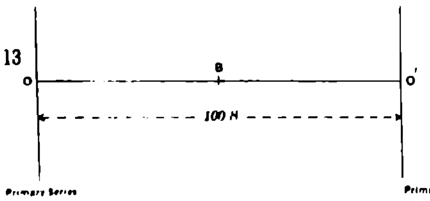
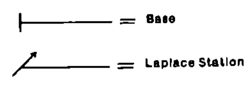


Fig. 13





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Helio S. I. O. Dehra Dun

So far as scale is concerned, M gives exactly the same results as para 14 except that it (*a*) takes no account of the ratio length ÷ breadth in elongated figures, and (*b*) derives the probable error of an observed angle from triangular closures only, and not from figural adjustments or other considerations.

For the probable azimuth error after a single figure M gives $1 \cdot 15\,e$ for pairs of triangles, $0 \cdot 99\,e$ for quadrilaterals and $1 \cdot 30\,e$ for hexagons. The difference between these figures and the $0 \cdot 75\,e$, $0 \cdot 55\,e$ and $0 \cdot 88\,e$ of para 15 arises from the fact that M is based on G.T. Vol. II, which treats all angles as independently measured. The probable error of a 180° angle is therefore taken to be $e\sqrt{3}$, whereas it has here been taken to be $1 \cdot 25 \times e$. Para 19 shows that circuit and Laplace closures require the figures of para 15 to be multiplied by an empirical factor of $1 \cdot 5$, when they become $1 \cdot 13\,e$, $0 \cdot 83\,e$ and $1 \cdot 32\,e$ respectively, and so agree quite closely with the figures derived from M.

17. **Accumulation of scale and azimuth error.**—It is convenient to consider the accumulation of error in 100 miles of series. Suppose a series contains n_1 figures* per 100 miles, and n_2 flank sides (the mean of both flanks). Then N_1, the probable error of log side after 100 miles, will be $ABe\sqrt{n_1}$ where :—

A is $24 \cdot 8$ for centered quadrilaterals and pairs of triangles
$21 \cdot 4$ for braced quadrilaterals
$27 \cdot 6$ for pentagons, hexagons and other centered figures.
A weighted mean value is taken for mixed series.

B is the mean value of length÷breadth for the figures of the series.

e is the probable error of an observed 60° angle.

Similarly, N_2 the probable error of azimuth after 100 miles is $Ce\sqrt{n_2}$, where :—

C is $0 \cdot 75 \times 1 \cdot 5\dagger = 1 \cdot 13$ for centered quadrilaterals and pairs of triangles.
$0 \cdot 55 \times 1 \cdot 5 = 0 \cdot 83$ for braced quadrilaterals
$0 \cdot 62 \times 1 \cdot 5 = 0 \cdot 93$ for pentagons, hexagons and other centered figures.

Values of N_1 and N_2 for all primary and secondary series are given in Appendix VI.

Then after a series whose length is $100S$ miles, the probable error is :—
(*a*) In scale $N_1\sqrt{S}$, in the 7th decimal of the log.
(*b*) In azimuth $N_2\sqrt{S}$ seconds.

And after several consecutive series, the probable error is
(*a*) In scale $\sqrt{\Sigma N_1^2 S}$
(*b*) In azimuth $\sqrt{\Sigma N_2^2 S}$

If the initial side of the series has a probable error of s in the 7th decimal of the log side and t'' in azimuth, the probable errors of the terminal side are
(*a*) In scale $\sqrt{s^2 + \Sigma N_1^2 S}$
(*b*) In azimuth $\sqrt{t^2 + \Sigma N_2^2 S}$

* A pair of triangles being one figure. A double centered figure counts as 1½ hexagons and a triple centered figure as two.

† The factor 1·5 is derived in para 19.

18. Accumulation of error of position.—In Professional Paper No. 16, pages 106 to 161, Dr. J. de Graaff Hunter has considered the probable error of position in adjusted and unadjusted circuits. The method finally arrived at requires the simultaneous solution of as many equations as there are base, Laplace and circuit conditions in the triangulation (e.g., 48 for the NW. Quadrilateral), and the reasons given in para 8 for avoiding least squares in the adjustment apply with even greater force to the determination of its probable errors.

Appendix VII gives results for some simple cases as shown in Plate VIII. The results are summarized below. In each case O is the origin relative to which the probable error is to be found, and A is the point at which it is required. E_1 is the probable error of position in feet in the direction OA, and E_2 is the probable error normal to OA. Distances are measured in units of 100 miles. (See list of symbols at the beginning of this Chapter).

The numerical factors theoretically found in Appendix VII have here been multiplied by 1·33 as the result of comparison with actual circuit closures given in para 19. N_2 is also presumed to have been computed with the factor 1·5 derived in the same para, as has been done in Appendix VI.

Case I. (Plate VIII, Fig. 1). Base and Laplace station at O, assumed errorless, and none at A. Triangulation straight.

At A $E_1 = 0 \cdot 093\ N_1 S\sqrt{S}$ feet

$E_2 = 1 \cdot 97\ \ N_2 S\sqrt{S}$ feet

Case II. (Plate VIII, Fig. 2). The same as Case I, but the triangulation between O and A consists of several series of length 100 L_1, 100 L_2 miles etc., not lying in a straight line

At A $E_1 = 0 \cdot 162\sqrt{\Sigma N_1^2 L R^2}$ feet

$E_2 = 3 \cdot 41\ \sqrt{\Sigma N_2^2 L R^2}$ feet

where 100 R miles is the distance from A to the centre of each series.

Note.—If the separate series are all of the same strength, this result is closely reproduced by using Case I with S = mean of direct distance OA and distance following the triangulation.

Case III. (Plate VIII, Fig. 5). Bases and Laplace stations at both O and A, assumed errorless. Triangulation straight.

At A $E_1 = 0 \cdot 047\ N_1 S\sqrt{S}$ feet

$E_2 = 0 \cdot 98\ \ N_2 S\sqrt{S}$ feet

Notes.—(1). Comparing with Case I, the base and Laplace station at A are seen to have halved the error.

(2). At B, the middle point of the series, the probable errors are 0·56 of those at A.

(3). At B, the probable errors are 0·79 of what they would have been by Case I if the series OB stood alone. The probable errors of points nearer O are thus approximately given by Case I.

(4). See Fig. 6. If instead of being at A, the base and Laplace station are connected to it by a series, straight or curved, of strength and length equal to those of OA, the probable errors at A are

$E_1 = 0 \cdot 074\ N_1 S\sqrt{S}$ feet

$E_2 = 1 \cdot 56\ \ N_2 S\sqrt{S}$ feet

As in note (3), these figures are 0·79 of those of Case I.

Case IV. (Plate VIII, Fig. 7). The same as Case II, but with bases and Laplace stations, assumed errorless, at both O and A.

$$\text{At A} \quad E_1 = 0\cdot162\sqrt{\Sigma N_1^2 L\ (\rho^2 + L^2/12)}\ \text{feet}$$
$$E_2 = 3\cdot41\ \sqrt{\Sigma N_2^2 L\ (\rho^2 + L^2/12)}\ \text{feet}$$

where 100ρ miles is the distance from the centre of each series to the centre of gravity of the whole. If the series are of unequal strength, the centre of gravity is determined after weighting each in proportion to its N^2.

Note :—If the separate series are all of the same strength, this result is closely reproduced by using Case III with S = mean of direct distance OA and distance following the triangulation.

Case V. (Plate VIII, Fig. 8). Two bases and Laplace stations, assumed errorless, are joined by two series in parallel.

$$E_1 = 1/\sqrt{1/E^2_{11} + 1/E^2_{12}} = E_{11}\ E_{12}/\sqrt{E^2_{11} + E^2_{12}}$$
$$E_2 = 1/\sqrt{1/E^2_{21} + 1/E^2_{22}} = E_{21}\ E_{22}/\sqrt{E^2_{21} + E^2_{22}}$$

Second suffices 1 and 2 refer to the two different routes.

Case VI. Measured base-lines have probable errors s in the 7th figure of the log, and Laplace azimuths have a probable error of t''.

(*a*). Then in Cases I and II an error $0\cdot122sS$, must be combined (root sum of squares) with E_1 as given above, and $2\cdot56tS$ must be combined with E_2. (In case II 100 S is the straight distance OA).

The formulæ of Case I become

$$E_1 = 0\cdot093\ S\sqrt{N_1^2 S + 3u^2}\ \text{feet}$$
$$E_2 = 1\cdot97\ \ S\sqrt{N_2^2 S + 3v^2}\ \text{feet}$$

where $u = 0\cdot75\,s$ and $v = 0\cdot75\,t$.

This factor $0\cdot75$ arises from the fact that the augmenting factor of $1\cdot33$ which has been applied to terms such as N_1^2S, is not required in terms arising from s and t.

(*b*). In cases III, IV and V, the errors which have to be combined with E_1 and E_2 are $0\cdot061\ S\sqrt{s_1^2 + s_2^2}$ and $1\cdot28\,S\sqrt{t_1^2 + t_2^2}$, where s_1 and s_2 refer to the two bases. If $s_1 = s_2$ and $t_1 = t_2$, the formulæ of Case III become

$$E_1 = 0\cdot047\,S\sqrt{N_1^2 S + 6u^2}\ \ \text{feet}$$
$$E_2 = 0\cdot98\ \ S\sqrt{N_2^2 S + 6\,v^2}\ \ \text{feet.}$$

Case VII. (Plate VIII, Fig. 9). Two series OB and BA of lengths $100\,S_1$ and $100\,S_2$ miles lie in a straight line with bases and Laplace stations at O, B, and A. The latter have probable errors of $s_1\ s_2\ s_3$ and $t_1\ t_2\ t_3$ respectively. N_{11} and N_{21} apply to OA, and N_{12} and N_{22} to AB.

$$\text{At A} \quad E_1 = 0\cdot047\sqrt{N^2_{11}\ S_1^3 + N^2_{12}\ S_2^3 + 3u_1^2\ S_1^2 + 3u_2^2\ (S_1 + S_2)^2 + 3u_3^2\ S_2^2}\ \text{feet}$$
$$E_2 = 0\cdot98\ \sqrt{N^2_{21}\ S_1^3 + N^2_{22}\ S_2^3 + 3v_1^2\ S_1^2 + 3v_2^2\ (S_1 + S_2)^2 + 3v_3^2\ S_2^2}\ \text{feet}$$

Notes.—(1) If $N_{11} = N_{12} = N_1$, $S_1 = S_2 = S$, $s_1 = s_2 = s_3$ etc., these become

$$E_1 = 0\cdot047\ S\sqrt{2\,N_1^2\,S + 18u^2}\ \text{feet}$$
$$E_2 = 0\cdot98\ \ S\sqrt{2\,N_2^2\,S + 18v^2}\ \text{feet}$$

(2) In general, if the E_1 of OB is E_{11} (obtained by cases III, IV or V) and that of BA is E_{12}

$$E_1 = \sqrt{E^2_{11} + E^2_{12} + (0\cdot061)^2\ \{s_1^2\ S_1^2 + s_2^2\ (S_1 + S_2)^2 + s_3^2\ S_2^2\}}\ \text{feet.}$$

Similar formulæ apply for E_2, and for the case of three series in line. See Appendix VII.

(3) If there is a right-angle bend at B, the errors of the triangulation series OB and BA can be combined (root sum of squares) without considering this case.

Case VIII. (Plate VIII, Fig. 10). A series OA of length 100 (S_1+S_2) miles with base and Laplace stations at either end, is met at B $(OB=S_1)$ by a short series BC of length 100 S_3 miles, with a base and Laplace station at C. The probable errors of all bases and Laplace stations are s and t, and N_1 and N_2 apply to all three series.

Then apply Case VII, replacing the series BC by a base and Laplace station at B having probable errors of $\sqrt{\dfrac{N_1{}^2S_1S_2S_3}{S_1S_2+S_2S_3+S_3S_1}+s^2}$ and $\sqrt{\dfrac{N_2{}^2S_1S_2S_3}{S_1S_2+S_2S_3+S_3S_1}+t^2}$ respectively.

If the strength of BC is not represented by N_1 and N_2 replace S_3 by $S_3\ N^2{}_{BC}/N^2{}_{OA}$.

Note.—These expressions give s and t. Before substituting in the formulæ of Case VII (except those of Note 2), they must be multiplied by $0\cdot75$ to give u and v.

Case IX. (Plate VIII, Fig. 11). See Appendix VII. In AA′, s and t as obtained by para 17 must be multiplied by $0\cdot75$ to give u and v.

Case X. (Plate VIII, Fig. 12). Two secondary series OA and O′A, each of length $100S$ miles and of equal strength, lying parallel and close to each other, emerge from a primary series OO′ (assumed errorless), and are connected together at A.

At A $\quad E_1=0\cdot052\ N_1\ S\sqrt{S}$ feet

$E_2=1\cdot10\ \ N_2\ S\sqrt{S}$ feet

Notes.—(1). If the scale and azimuth of the primary series near O and O′ have probable errors of s and t, the above formulæ become

$$E_1=0\cdot052\ S\sqrt{N_1{}^2S+5\cdot3s^2}\ \text{feet}$$

$$E_2=1\cdot10\ \ S\sqrt{N_2{}^2S+5\cdot3t^2}\ \text{feet}$$

(2). The total probable relative error of B and B′, the middle points of the two secondary series, is

$$E=0\cdot033\ NS\sqrt{S}\ \text{feet.}$$

Case XI. (Plate VIII, Fig. 13). A secondary series OO′ of length $100S$ miles lies between two primary series, assumed errorless. At B, its middle point

$$E_1=0\cdot018\ N_1\ S\sqrt{S}$$

$$E_2=0\cdot39\ \ N_2\ S\sqrt{S}$$

$$E\ =0\cdot018\ N\ S\sqrt{S}$$

Note.—If the scale and azimuth of the primary series are fallible, see Appendix VII. In formulæ (20*a*) to (20*e*) substitute u and v for s and t, and substitute factors $0\cdot047$ and $0\cdot98$ for $0\cdot035$ and $0\cdot74$ respectively.

19. Application to Indian circuit closures.—If triangulation connects two base-lines, the probable misclosure in scale is clearly $\sqrt{2s^2 + \Sigma N_1^2 L}$, where 100 L miles is the length of each series which requires a separate value of N_1, and s is the probable error of each base-line and its extension. Similarly, the probable misclosure between two Laplace stations is $\sqrt{2t^2 + \Sigma N_2^2 L}$, where t is the probable error of the azimuth correction deduced at a Laplace station. The probable misclosures of a circuit, ignoring base-lines and Laplaces stations, are clearly $\sqrt{\Sigma N_1^2 L}$ and $\sqrt{\Sigma N_2^2 L}$ respectively.

Appendix VIII considers the values which should be assigned to s and t, and concludes that $s = 12$ in the 7th decimal of the log, and $t = 0'' \cdot 5$. In the later part of this paragraph it is concluded that the value of t should be increased to $0'' \cdot 75$.

Table 3 gives the calculated probable misclosures between the Indian base-lines by the shortest and strongest routes. It also gives the actual misclosures and the ratios of actual to probable. The distribution of the 21 different values of the ratio is given in Table 2, line 1. Table 4 and Table 2, lines 2 and 3, give similar data for the 39 Laplace closures*. Table 5 gives the probable and actual misclosures in scale, azimuth and position at the 32† circuit closures (marked by arrow points on Plate I), and the distribution of the ratios actual to probable is shown in Table 2, lines 4, 5, and 11.

Table 2, line 6, gives the distribution of the ratios for all scale misclosures, base-line or circuit, and line 7 gives the normal distribution of 52 such ratios according to the theory of probability. The agreement between lines 6 and 7 is good. These two lines show that, if anything, the actual misclosures are slightly smaller than those calculated, but the difference is not significant, and it may be concluded that the values of N_1 which have been assigned in Appendix VI do on the average accurately indicate the ability of the primary series to carry forward the scale.

Table 2, line 8, similarly gives the distribution of the ratios for all azimuth misclosures, and line 9 gives the normal distribution of 70 ratios. Here there is a serious discrepancy, for the actual ratios tend to have many more high values than the normal. This discrepancy may be due to either error in N_2 or in t. If N_2 is too low the high values of the ratio will tend to occur on long lines, whereas if t only is too low the high values will occur especially in the misclosures between near Laplace stations, and not at all in the circuit misclosures. Line 3 shows the distribution of the ratios for close Laplace stations. It is very similar to line 8, and suggests that the fault lies equally in N_2 and in t. Line 10 shows the result of multiplying the calculated probable values by $1 \cdot 5$, and this gives good agreement with line 9. It is therefore concluded that the values theoretically obtained for probable azimuth error in para 15, and the value of t obtained in Appendix VIII, require to be multiplied by $1 \cdot 5$. This factor has been accepted in paras 17 and 18, and in the values of N_2 given in Appendix VI, which therefore on the average accurately indicate the ability of the primary series to carry forward azimuth.

That the misclosures in scale should on the average agree with theory, while those in azimuth do not, requires explanation. The most probable explanation seems to be that the probable error of a 180° angle is more than the $1 \cdot 25$ times that of a small angle, which was deduced from internal evidence only in Appendix V. It appears that $1 \cdot 87$ times is nearer the correct figure.

* Ignoring the factor of $1 \cdot 5$ in the computation of N_2 and taking $t = 0'' \cdot 5$.
† One point, 45, gives no scale or azimuth closure.

That internal evidence should have failed to give an accurate result is not at all surprising. It is notable, but presumably due only to chance, that 1·87 is similar to the $\sqrt{3}$ or $\sqrt{4}$ which would be applicable if all angles were independently measured.

Table 2, line 12, gives the normal distribution of 32 ratios of actual to probable misclosure in position, for comparison with line 11. Here also the actual ratios have a higher average than the probable, showing that the formulæ of Appendix VII, Case II, give too low a value for the closing error of position, although they employ values of N_1 and N_2 which correctly give the probable errors in scale and azimuth. An augmenting factor of 1·33 secures the correct distribution as shown in Table 2, line 13, and this augmenting factor must similarly be applicable to all the formulæ of Appendix VII*. It is included in the summary of formulæ given in para 18 of this Chapter.

The cause of this discrepancy lies perhaps in the approximation, employed throughout Appendix VII, that $\sqrt{(1^2+2^2+\ldots n^2)} = \sqrt{n^3/3}$, where n is the number of figures in the triangulation. If $n=10$, the error in this assumption is only 7%, while an error of 33% occurs only if n is as small as 4 or 5. The number of figures in any circuit is generally very much more than 10, and the error on this account should be negligible, but in triangulation it is inevitable that the quality should be uneven. After a number of good figures, some unfavourable condition occurs which results in an exceptionally bad figure, and it is possible that these occasional bad figures (numbering perhaps 4 or 5 per circuit) are responsible for most of the error, and so for the necessity for an augmenting factor of 1·33.

20. Probable errors of adjusted triangulation.—Although the Indian triangulation, as a whole, forms a very complicated network, many of the series are of relatively low quality, and the application of the formulæ of para 18 to a few main series forming a comparatively simple network will give results which would not be much changed if all the weaker series could be incorporated.

Thus the main framework of the triangulation is the Great Arc running meridionally from Dehra Dūn to Cape Comorin, with the Karāchi and Calcutta longitudinal series crossing it at Kaliānpur, the origin of the Survey. Cases III, VI and VII of para 18 at once provide the means of writing down preliminary values of the probable errors of position at Dehra Dūn, Cape Comorin, Karāchi and Calcutta, and it only remains to decide on the amount by which parallel series will reduce these figures, and to extend the procedure by similar methods to the limits of the area triangulated. Appendix IX gives details of the procedure followed. Final results are given in Plate IX, which shows the probable errors of position relative to the origin at Kaliānpur in the primary series as now adjusted†.

In the above, no account is taken of error in the standard of length in terms of which all distances have been expressed. The ten old base-lines on which India proper is almost entirely dependent were expressed in terms of tenths of the standard 10-foot Bar A, which was standardized by Clarke in 1865, and this unit (the Indian foot) has been retained. That it does not

* There is no reason to apply the augmenting factor to terms involving s and t in these formulæ. See para 18.

† There is at present a weakness east of Calcutta. Assam and Burma are not connected to India by any primary series. It is hoped to remedy this in 1939-40-41. The figures given in Plate IX are those which will be applicable when this has been done. In the mean time, positions in Assam and Burma are in doubt by perhaps a further 20 feet.

exactly equal the British foot is of no consequence, but there is inevitably some doubt about what the true length of Bar A was at the time when the base-lines were measured. Details are given in Appendix X where it is concluded that a doubt of 3 parts in a million is possible. This figure is thought to represent the maximum possible error, not the probable. The seven modern base-lines do not share this possible error since modern standards have been used for them. Plate IX shows (in red) the error of position relative to Kaliānpur, which would arise from this maximum error of 3 in 10^6 in India, combined with an error of 1 in 10^6 (if of the same sign) in the modern standards which have been used in Baluchistān, Assam and Burma. It is seen that this maximum error on account of doubt in the standards is about equal to the probable error generated in the triangulation, so that the former probably contributes little to the total error arising from all causes, and is not referred to further.

Plate IX gives errors relative to Kaliānpur. The relative errors of any two points whose strongest connection is via Kaliānpur, can be obtained by combination of the figures there given. Thus the probable relative error of Kūh-i-Malik Siāh (point 14 in Plate 1) on the Irān frontier and Kēng Tung (86) near the Indo-Chinese frontier is $\sqrt{17 \cdot 8^2 + 17 \cdot 4^2} = 25$ feet or 1 in 530,000 of the distance between them. The probable relative error of Dehra Dūn (27) and Cape Comorin (61) is similarly 1 in 440,000. A probable error of about 1:500,000 may thus be said to represent the accuracy with which the triangulation is capable of determining the mean curvature of the geoid in India.

Plate X shows the probable errors of position of secondary triangulation relative to the nearest primary triangulation, ignoring any errors of position of the latter, but allowing for the fact that the primary triangulation is fallible as regards scale and azimuth. The method of calculation is given in Appendix IX. It also shows the probable relative errors of several long and closely parallel series in northern India, which from the topographical point of view constitute the weakest part of the triangulation.

Table 6 gives the probable errors of scale, azimuth and position relative to Kaliānpur at points where junctions have been, or may be, made with foreign surveys. A satisfactory degree of accuracy is shown at the four primary points where connection has been made with Ceylon and Siam (Latitudes 10° and 20°), and will eventually be made with Irān, but the secondary connections with Russia and Siam (in latitude 14°) are very weak.

The probable errors of scale and azimuth in the primary triangulation may be judged from Tables 3 and 4. The average closing errors on base-lines and Laplace stations are 47 (in the 7th decimal) and $2'' \cdot 3$ respectively. The probable errors at the centres of the series directly connecting these controls are obtained by multiplying these figures by $\frac{1}{2} \times 0 \cdot 84$, and are consequently 20 and $0'' \cdot 9$. In Table 1 A the errors of scale and azimuth of adjusted primary series are thus very unlikely to exceed 60 and $2\frac{1}{2}''$, except in a few places such as the Sutlej and Gurhāgarh series (23–20–19) which are exceptionally far removed from base-lines. In a few such places the scale error may possibly be 100. Similarly there are a few places such as the South-east Coast series (62–64) and NW. of Calcutta (38–39) where azimuth errors change rapidly, and adjusted azimuths may possibly be wrong by 3″ or 4″.

The triangulation in Assam north-east of Calcutta is also weak. The provisional values given in Table 1B may have larger errors than 60 and 2½″, both there and in parts of Burma.

The largest errors in secondary series will very seldom, if ever, exceed 200 and 7″, except in the Indo-Russian and Siam Branch pendant series whose probable errors are given in Table 6.

The largest possible error quoted above amounts to 1:20,000 and both primary and secondary triangulation can be regarded as providing values of scale and azimuth which are errorless from the point of view of topographical and cadastral work, provided pairs of stations can be found whose mark-stones can be trusted to have undergone no relative movement of this amount. See para 27.

The figures given above refer to the triangulation as now readjusted. Errors in azimuth of 12″ (i.e., 1:17,000) are known to occur in Burma in the figures at present published, but this does not prevent their being regarded as errorless for non-geodetic purposes.

TABLE 2.—*Distribution of ratios of actual to probable errors.*

		Number of cases with values between							Total number of cases
		0—1	1—2	2—3	3—4	4—5	5—6	>6	
1	Base-line closures ...	13	5	2	1	...	...	...	21
2	Laplace closures ...	11	13	5	6	...	2	2	39
3	Laplace closures (close only)*	3	3	2	3	...	...	1	12
4	Circuit closures. Scale ...	19	9	2	1	...	...	...	31
5	Circuit closures. Azimuth ...	6	11	8	2	2	1	1	31
6	Line 1 *plus* line 4 ...	32	14	4	2	...	...	...	52
7	Normal distribution ...	26	17	6	3	...	...	...	52
8	Line 2 *plus* line 5 ...	17	24	13	8	2	3	3	70
9	Normal distribution ...	35	23	9	2	1	...	...	70
10	As line 8 after multiplying probables by 1·5 ...	33	21	9	4	3	...	...	70
		0—0·5	0·5—1·0	1·0—1·5	1·5—2·0	2·0—2·5	2·5—3·0	>3·0	
11	Circuit closures. Position ...	1	6	14	5	4	...	2	32
12	Normal distribution† ...	5	11	9	5	1	1	...	32
13	As line 11 after multiplying probables by 1·33 ...	3	14	8	5	1	1	...	32

* i.e., within 100 miles.

† The normal distribution of errors over an area is given by the formula:—
Percentage of errors less than a/a_0 is $100\left(1-e^{-0\cdot 693(a/a_0)^2}\right)$
where a_0 is the probable error, and a is any specified magnitude.

TABLE 3.—*Base-line closures.*

In units of the 7th decimal of the log.

Between Bases	Route	Calculated probable error	Actual error	Actual ÷ probable
Sironj-Dehra Dūn ...	1—26—27	49	57	1·16
Dehra Dūn-Chach ...	27—30—25	38	52	1·37
Sironj-Karāchi ...	1—3—6—8	51	69	1·35
Karāchi-Chach ...	8—10—23—24—25	42	166	3·96
Padag-Chach ...	12—16—24—25	35	56	1·60
Padag-Karāchi ...	12—15—9—8	32	31	0·97
Sironj-Calcutta ...	1—36—38—39	28	58	2·07
Sironj-Bider ...	1—46—47	35	22	0·63
Calcutta-Vizagapatam ...	39—41—43	46	19	0·41
Bider-Vizagapatam ...	47—49—43	25	12	0·48
Bider-Poona ...	47—52—51	49	25	0·51
Bider-Bangalore ...	47—54	27	19	0·70
Bangalore-Cape Comorin ...	54—59—61	28	1	0·04
Sironj-Poona ...	1—4—51	73	47	0·64
Calcutta-Sonākhoda ...	39—71	101	24	0·24
Calcutta-Nāmtiāli ...	39—69—70—79—74—75	126	46	0·36
Nāmtiāli-Kalemyo ...	75—74—79—88	61	66	1·08
Nāmtiāli-Kēng Tung ...	75—76—81—84—86	41	15	0·37
Kēng Tung-Amherst ...	86—95—93—98	51	126	2·48
Kalemyo-Amherst ...	88—91—93—98	61	48	0·79
Amherst-Mergui ...	98—99—101	60	22	0·37

TABLE 4.—*Laplace closures.*

Between Laplace Stations	Route	Calculated probable error	Actual error	Actual ÷ probable	REMARKS
Kaliānpur–Dehra Dūn ...	1—26—27	1″·60	3″·4	2·12	
Kaliānpur–Birona ...	1—3—5	1·06	0·1	0·09	
Birona–Chūtli ...	5—6—7	1·19	0·6	0·50	
Chūtli–Karāchi ...	7—8	0·88	1·8	2·05	Close
Karāchi–Yūsuf ...	8—10—17	0·93	0·7	0·75	
Birona–Vijnot ...	5—6—18	1·26	0·5	0·40	
Kaliānpur–Garinda ...	1—3—19	1·48	2·4	1·62	
Karāchi–Tozghi ...	8—10—12—13	0·96	2·3	2·40	
Kaliānpur–Dāmargīda etc.* ...	1—47	1·13	2·2	1·95	
Kaliānpur–Karaundi ...	1—35	0·77	0·8	1·04	
Karaundi–Tilabani ...	35—37—38	0·87	0·8	0·92	
Tilabani–Madhpur ...	...	0·75	0·3	0·40	Close
Madhpur–Calcutta ...	...	0·75	4·7	6·28	Close
Dāmargīda etc.–Vizagapatam etc.	47—49—43	0·90	2·8	3·11	
Calcutta–Vizagapatam etc. ...	39—41—43	1·41	1·5	1·06	
Dāmargīda etc.–Māndvi etc. ...	47—52—51	1·38	1·6	1·16	
Dāmargīda etc.–Bangalore ...	47—54	0·95	4·8	5·06	
Māndvi etc.–Mangalore ...	52—53	1·24	1·9	1·53	
Mangalore–Nughallibětta ...	...	0·76	2·7	3·55	Close
Nughallibětta–Bangalore ...	...	0·76	1·2	1·58	Close
Bangalore–Anandalamalai etc.	54—55	0·86	0·4	0·47	
Bangalore–Kutipārai ...	54—59—60	0·92	1·4	1·52	
Kutipārai–Kudankulam etc. ...	60—61	0·74	0·9	1·22	Close
Anandalamalai etc.–Kallapat	55—62	0·79	1·1	1·39	Close
Kallapat–Pātharankota ...	62—63	1·00	5·6	5·60	
Pātharankota–Manĕgandi ...	...	0·79	2·9	3·68	Close
Manĕgandi–Ramnad ...	...	0·79	0·4	0·51	Close
Ramnad–Kutipārai ...	64—60	0·94	0·8	0·85	Close
Calcutta–Daulatpur ...	39—67	2·40	8·4	3·50	Close
Daulatpur–Lakhinagar ...	67—68	1·91	3·9	2·04	Close
Lakhinagar–Semu Tan ...	68—87	1·95	3·1	1·59	
Semu Tan–Dat Taung ...	87—91	0·97	3·1	3·20	
Dat Taung–Taungzun ...	91—93—98	1·16	7·7	6·64	
Semu Tan–Tatalia ...	87—69—70—79—74—73	1·76	2·0	1·14	
Tatalia–Nāginimāra ...	73—74—75	1·02	1·4	1·37	
Nāginimāra–Mingun ...	75—76—81—90	1·04	0·5	0·48	
Mingun–Kĕng Tung ...	90—94—95—86	1·11	3·8	3·42	
Mingun–Taungzun ...	90—93—98	1·07	2·9	2·71	
Taungzun–Mergui ...	98—99—101	2·48	1·1	0·44	

* Groups of Laplace stations are here referred to by the name which is given first in Appendix II, Table 10.

NOTE:—The probable errors have been calculated with the factors 0·75, 0·55 and 0·62 of para 15, without the augmenting factor of 1·5 derived in para 19.

TABLE 5.—*Circuit closures.*

E = Calculated probable error   A = Actual error.

At	Scale in 7th decimal of log			Azimuth in seconds			Position in feet		
	E	A	A/E	E	A	A/E	E	A	A/E
29	61	92	1·51	1·9	6·7	3·53	30	43	1·43
30	78	125	1·60	2·2	1·5	0·68	41	43	1·05
31	51	151	2·96	1·6	4·2	2·62	10	24	2·40
25	54	5	0·09	1·6	3·0	1·88	19	25	1·31
22	75	14	0·19	2·1	2·8	1·33	25	33	1·32
18	45	51	1·13	1·3	1·4	1·08	14	22	1·57
23	89	101	1·13	2·5	3·2	1·28	36	46	1·28
12	33	56	1·70	0·8	1·7	2·12	6	7	1·16
24	91	78	0·86	2·5	3·2	1·28	46	30	0·65
49	41	71	1·73	1·1	0·2	0·18	17	22	1·29
50	35	32	0·91	1·0	5·0	5·00	12	29	2·41
43	35	2	0·06	1·0	1·0	1·00	11	12	1·09
45*	...	...	...	...	...	...	13	16	1·23
42	46	32	0·70	1·3	2·9	2·23	18	26	1·44
48	52	20	0·38	1·6	2·5	1·56	13	5	0·38
55	39	4	0·10	1·1	4·6	4·18	13	24	1·84
60	52	21	0·40	1·4	9·0	6·43	13	50	3·84
54	48	30	0·62	1·5	3·6	2·40	25	30	1·20
57	48	15	0·31	1·4	3·6	2·57	24	20	0·83
62	65	5	0·08	1·9	6·5	3·42	6	8	1·33
63	60	185	3·08	1·7	3·9	2·29	8	16	2·00
51	90	44	0·49	2·6	6·6	2·54	30	47	1·56
72	125	230	1·84	3·7	2·4	0·65	32	18	0·56
73 (Left)	100	102	1·02	2·9	13·1	4·52	28	59	2·11
91	76	16	0·21	2·0	1·1	0·55	23	35	1·52
88	61	29	0·48	1·5	1·3	0·87	10	23	2·30
90	151	189	1·25	2·8	3·6	1·29	17	26	1·53
93	69	43	0·62	1·6	3·3	2·06	36	43	1·19
86	53	136	2·57	1·1	1·3	1·18	11	34	3·09
73 (Right)†	61	19	0·31	1·5	3·9	2·60	12	9	0·75
82	68	35	0·51	1·5	0·4	0·27	11	8	0·73
81	56	16	0·29	1·4	1·5	1·07	9	8	0·89

* No common side.   † 1934–35 values. Observations of 1937–38 provide revised values.

NOTE:—The probable errors in azimuth have been computed with the factors 0·75, 0·55 and 0·62 of para 15, without the augmenting factor of 1·5 derived in para 19. This augmenting factor has been incorporated in N_1 before calculating the probable error of position, but the second augmenting factor of 1·33 derived for position in para 19 has not been included. The formula used for the probable error of position is thus that given in Case II of Appendix VII, and not that given with the augmenting factor in para 18.

TABLE 6.—*Probable errors of the Indian triangulation at junctions with foreign surveys.*

Junction Side	Reference on Plate I	Scale	Probable error of azimuth	Position	Junction with	Quality of Indian Triangulation
			"	*feet*		
Kūh-i-Malik Siāh–Kācha Kūh ...	14	0·0000026	0·8	18	Irān (future)	Primary
Kachi Tīvu North–Kachi Tīvu South ...	65	*	1·0	14	Ceylon	Primary
Loi Pakulin–Loi Tum ...	95	18	1·0	17	Siam	Primary
Khao Jamya–Khao Natathern ...	103	22	1·0	20	Siam	Primary
Khao Luang–Khao Ang Hin ...	100	122	5·0	21†	Siam	Weak Secondary
Sar-bulak–Kok-tek ‡ ...	34	560	9·6	72	Russia	Weak Secondary

* The side is so short that no useful value of scale can be carried through it.

† This figure is little greater than the probable error at the two primary junctions with Siam, since the probable error of position generated in the 70 miles of the weak Siam Branch series is less than that generated in the 1,700 miles from Kaliānpur. The former amounts to 11 feet.

‡ There is a Russian base-line at Kizil Rabāt, close to Sar-bulak, on which the Indian triangulation closes with an error of 930 in the 7th decimal of the log. The figures given in Tables 1 A and 6 and in Plate X refer to Indian work only, and ignore this base-line. A base-line at Kizil Rabāt should multiply the probable error of position at Sar-bulak by about 0·7, but in view of the actual closing error in log side being nearly twice the probable it is doubtful whether this reduction is justifiable.

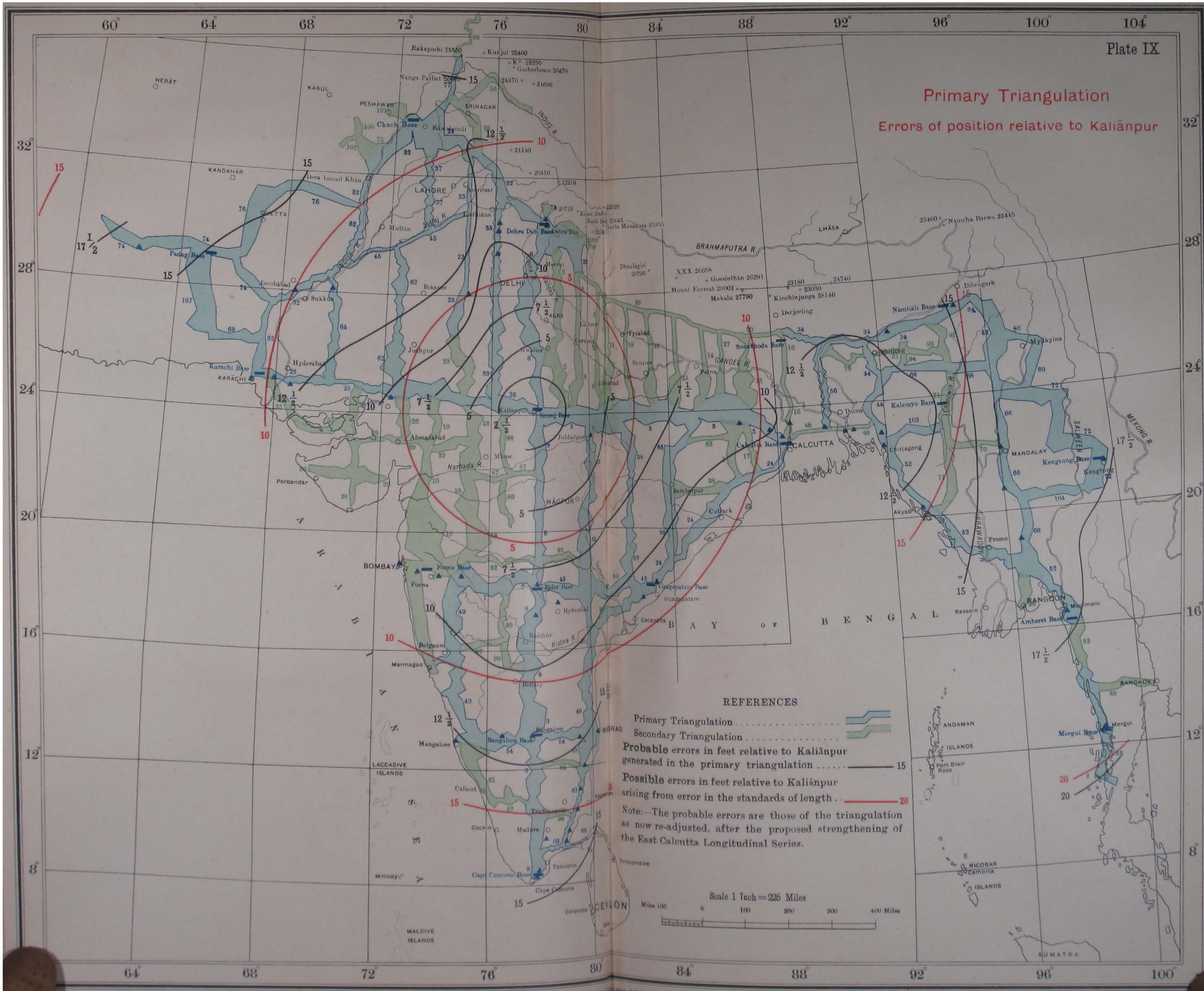

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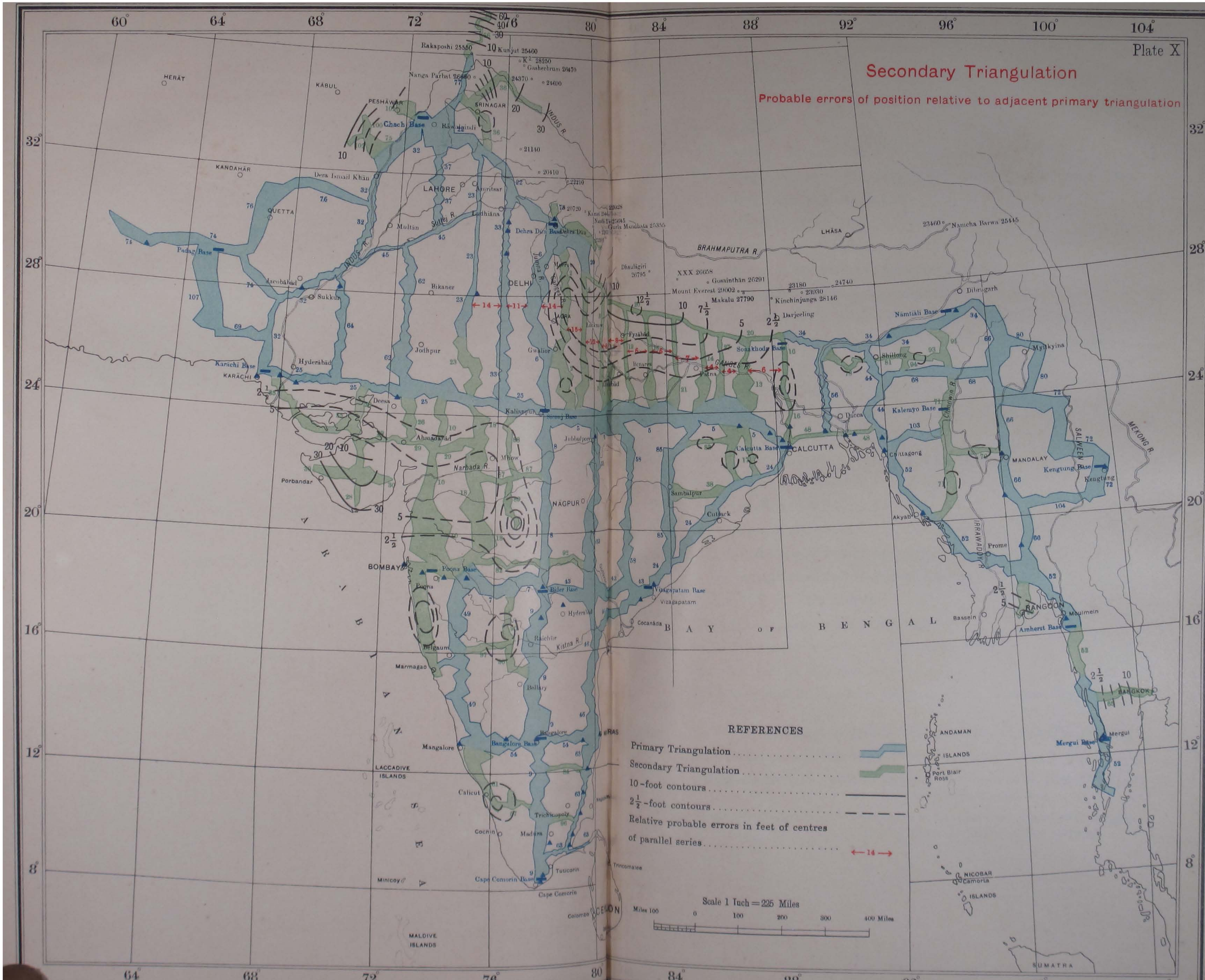
Plate X



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SYMBOLS USED IN CHAPTER III

ϕ = Latitude.

ϕ' = Reduced latitude. $\tan\phi' = (1-\epsilon)\tan\phi$.

λ = Longitude, measured eastwards from the origin. But see $\delta\lambda$ below.

a = Semi major axis of spheroid.

ϵ = Ellipticity of spheroid $= (a-b)/a$.

e = Eccentricity of spheroid. $e^2 = 2\epsilon$.

ρ = Radius of curvature of spheroidal meridian.

ν = Normal terminated by the minor axis.

η = Deviation of the vertical in meridian. (South positive).

ξ = Deviation of the vertical in prime vertical. (West positive).

θ = Any other of the component of the deviation.

θ_1 = Inclination between two spheroids at any point } see para 25

θ_2 = Deviation of the vertical on a well-fitting spheroid } see para 25

S_1 = An old spheroid (e.g., Everest's).

S_2 = A new spheroid (e.g., the International).

N = Height of S_2 above S_1.

l = Length of a triangulation side.

α = Angle of elevation.

h, h_1, h_2 = Heights of triangulation stations.

β = Azimuth measured clockwise from south.

P, Q, R, T, U, V. See para 23.

Suffix zero indicates value at the origin.

Prefix δ indicates value for S_2 *minus* value for S_1.

$\delta\phi$ = Latitude on S_2 *minus* latitude on S_1.

$\delta\lambda$ = Longitude on S_2 *minus* longitude on S_1, measured from Greenwich.

$\delta\eta_0$ and $\delta\xi_0$ = Changes in deviation at the origin, expressed in radians.

$\Delta\lambda$ = Difference of longitude between two stations.

"Height correction" = Those terms of the formulæ of para 23 (a) to (d) which involve h, h_1, and h_2.

"Deviation correction" = Correction to observed horizontal angles, arising from deviation of the vertical.

Chapter III

CHANGE OF REFERENCE SPHEROID

21. The reference spheroid.—In the ordinary process of triangulation a theodolite set up at a station A measures the angle between other stations B and C as projected on to a plane perpendicular to the vertical axis of the theodolite. Provided the instrument is properly levelled, this plane is the tangent to the earth's equipotential surface which passes through the instrument, and is very closely parallel to the tangent to the geoid, or sea-level equipotential, at a point vertically below the instrument.

For the purpose of computation of latitudes and longitudes it is necessary to define a spheroid, clearly but arbitrarily* which throughout the area of the survey will lie reasonably close to the geoid. To each station of observation (A, B, or C) is related a point (A′, B′, or C′) vertically† below it. Provided the deviation of the vertical (the angle between geoid and spheroid) is known, it is solely a matter of computation to convert the measured angles BAC etc., to the spheroidal‡ angles B′A′C′. Similarly, if a horizontal distance AB is measured on the ground as a base-line, the distance A′B′ is immediately obtainable, provided the height above the spheroid is known. Then given (*a*) the distance A′B′, (*b*) the angle between A′B′, and the spheroidal meridian at A′, (*c*) all the spheroidal angles B′A′C′ etc., and (*d*) the spheroidal latitude and longitude of (say) A′, the spheroidal latitudes and longitudes of all points B′, C′ etc., can be computed, and these co-ordinates are ascribed to the stations of observation B, C etc.

22. Reduction of observations to spheroid level.—As compared with the cost of field observations computation is inexpensive, and so far as data are available computations can easily be kept more than sufficiently accurate. Unfortunately, necessary data are often lacking, namely the deviation of the vertical at each station of observation and the heights of base-lines above the spheroid.

Spirit-levelling and the vertical angles of ordinary triangulation give heights above the geoid, and a base-line computed with such a height is reduced to the geoid instead of to the spheroid. The separation between geoid and spheroid in different parts of a country can only be obtained by an elaborate series of astronomical observations covering the whole country§. This is now nearly complete in India, and Indian base-lines can be reduced to spheroidal level with sufficient accuracy ||.

A knowledge of the deviation of the vertical at each triangulation station is even less easily obtained. It involves astronomical observations at every station. This is a work which has not yet been undertaken in any country,

* Seven constants are required. The constants chosen must be convenient but are otherwise arbitrary. See Appendix XI.

† The word "vertically" requires careful definition. It is not here used in exactly its usual sense. See Appendix XII.

‡ The spheroidal angle at A′ is the angle between the spheroidal geodesics B′A′ and C′A′.

§ See any annual Survey of India Geodetic Report, e.g., 1936, Chapter III.

|| In a small country, with a well-chosen spheroid, the separation between geoid and spheroid may be negligible. In India, where Everest's spheroid is used, the separation may amount to 300 feet involving an error of 1 : 70,000 if neglected. See Appendix I.

with the result that the corrections arising from it have almost universally been ignored*. Generally the correction is very small. If the angle of elevation of B at A is a_B, and the component of the deviation of the vertical at right angles to AB is θ_B, the correction to the direction AB is $\theta_B \tan a_B$, so the correction to an angle BAC is $\theta_B \tan a_B - \theta_C \tan a_C$. If a_B and a_C are under 1° and if θ_B and θ_C are under 10″, the expression is the difference or sum of two terms each less than 0″·17. These conditions are quite normal, and except in so far as the corrections tend to remedy a systematic accumulation of error in scale or azimuth, the neglect of the deviation corrections is generally less serious than the effect of random errors of observation. Larger errors of course may sometimes occur, and an angle has actually been found in India where the correction amounts to 4½″, but corrections of more than 1″ are very rare. In any case there is no remedy other than expensive field work, and provided the systematic accumulation of scale and azimuth error is remedied by closure on correctly reduced base-lines and Laplace stations (see para 24), neglect of the deviation corrections, where not negligible, must simply be regarded as an addition to the ordinary casual errors of observation.

The fact that an astronomically observed azimuth must be corrected for the deviation of the vertical has long been known. The correction is included in the well-known Laplace's equation. It is only necessary to add that, for precise accuracy, it may also be necessary to include a correction $a \tan \theta$ to the direction of the station employed as referring mark, although this is generally of no more significance than the correction to any other angle.

23. Change of spheroid.—It may happen that some of the constants of the spheroid adopted when a system of triangulation is first computed subsequently come to be considered inconvenient. In the past different surveys have been computed on different spheroids, and as international agreement is reached regarding suitable axes, and as connection between different countries makes it possible to adopt spheroids with the same centres, it becomes desirable to change spheroid in order to secure uniformity. This applies with special force to India where the old Everest spheroid has been employed, whose axes differ by 3000 feet from those of more modern spheroids.

To recompute and readjust a large system of triangulation on a new spheroid is a very great labour, and some shorter method is required. Provided the triangulation has been correctly computed on its old spheroid (with reference to para 22) the change of spheroid is clearly a simple matter, involving little more than a knowledge of the angles between the tangents to the two spheroids at the point considered†.

Let A′, B′, C′ etc., be the points on an old spheroid S_1, corresponding to triangulation stations A, B, C (see para 21), and let A″, B″, C″ be the points corresponding to A, B, C on a new spheroid S_2. The relative positions of the two spheroids are specified by the following considerations.—

(*i*) Major axis of S_2 *minus* major axis of $S_1 = \delta a$

(*ii*) Ellipticity (or flattening) of S_2 *minus* that of $S_1 = \delta \epsilon$

* In India these corrections have been applied in a few places where their effects have been expected to be specially serious, such as in base-line extensions in hilly country.

† This was apparent to Dr. J. de Graaff Hunter when he was considering the question of change of spheroid in Professional Paper No. 16, pages 1 to 88, see page 77. The difference between his treatment and that now recommended arises from the fact that he was considering the effects of change of spheroid on the Indian triangulation as it was in 1918, without Laplace stations and with bases reduced to the level of the geoid. The effect of change of spheroid on the triangulation as now readjusted can be derived in a much more simple manner.

(*iii*) The minor axes of both spheroids are parallel to the earth's axis of rotation.

(*iv*) At the origin, the height of S_2 above $S_1 = N_0$

(*v*) The arbitrary latitude of the origin on S_2 *minus* that on $S_1 = \delta\phi_0$. At the origin $\delta\eta_0$, the meridional deviation* of the vertical on S_2 *minus* that on $S_1 = -\delta\phi_0$

(*vi*) The arbitrary Greenwich longitude of the origin on S_2 *minus* that on $S_1 = \delta\lambda_0$. At the origin $\delta\xi_0$, the prime vertical component of the deviation there on S_2 *minus* that on $S_1 = -\delta\lambda_0 \cos\phi_0$, where ϕ_0 is the latitude of the origin.

Then at any point, latitude ϕ and longitude λ (measured eastward from the origin as zero meridian), the height of S_2 above S_1, is given by†

$$N = P\,(U \sin\phi'_0 + V\cos\phi'_0) + Q\,\delta\xi_0 + R\,(V\sin\phi'_0 - U\cos\phi'_0) + \delta a - T\delta\epsilon$$

where $P = \cos\phi' \cos\lambda$

$Q = a\cos\phi' \sin\lambda$

$R = \sin\phi'$

$T = a\sin^2\phi'$

$U = -a\delta\eta_0 - a\delta\epsilon \sin 2\phi'_0$

$V = (N_0 - \delta a) + a\delta\epsilon \sin^2\phi'_0$

$\phi' =$ "reduced" latitude. $\tan\phi' = (1-\epsilon)\tan\phi$

$\delta\eta_0$ and $\delta\xi_0$ are expressed in radians.

Now consider the changes at any point B.

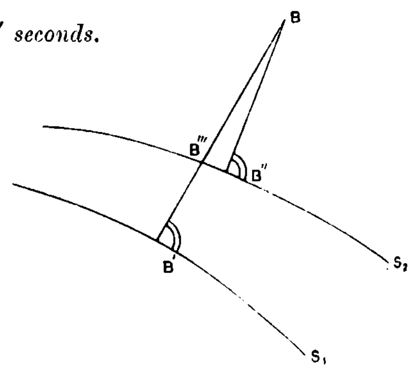
(*a*) If B is at sea-level, $\delta\phi$, the latitude on S_2 *minus* that on S_1, is the angle between the meridional tangents at B′ and B″, and is given by

$$\delta\phi = -\frac{1}{\rho}\,\frac{dN}{d\phi} = -\frac{1}{a}\left(1 + \epsilon\cos^2\phi\right)\frac{dN}{d\phi'}$$

$$= \left[1 + \epsilon\cos^2\phi\right]\ \left[\frac{1}{a}\sin\phi'\cos\lambda\left(U\sin\phi'_0 + V\cos\phi'_0\right)\right.$$

$$+ \sin\phi'\sin\lambda\,\delta\xi_0 - \frac{1}{a}\cos\phi'\left(V\sin\phi'_0 - U\cos\phi'_0\right)$$

$$\left. + \sin 2\phi'\,\delta\epsilon\right] \operatorname{cosec} 1'' \textit{ seconds.}$$



If B is at a height h above sea-level, the above ignores a term $B''B'''/\rho = -h\,\delta\phi/\rho$ (see marginal figure), and the expression for $\delta\phi$ must be multiplied by $(1 - h/a)$‡.

The change in deviation of the vertical in meridian $(\delta\eta) = -\delta\phi$.

* See Appendix XI. Southerly and westerly deviations are considered positive, i.e., when the inward normal of the geoid is south or west of the spheroidal inward normal.

† This formula has been given by Mr. B. L. Gulatee in Geodetic Report 1934, page 142. For convenience the signs of $\delta\eta_0$ and $\delta\xi_0$ have now been reversed to indicate values on S_2 *minus* values on S_1 instead of S_1 *minus* S_2 as was used there.

‡ $\delta\phi$ will never exceed 20″ so the correction $h\,\delta\phi/a$ will only be 0″·01 (or one foot) for a station 10,000 feet high. It is unnecessary to specify whether h is measured above S_1, S_2, or the geoid, and the similar small term $N\,\delta\phi/a$, which arises when B is at sea-level can always be ignored.

(*b*) Similarly $\delta\lambda$, the longitude on S_2 *minus* that on S_1 is given by *

$$\delta\lambda = -\frac{1}{\nu} \sec^2\phi \frac{dN}{d\lambda}\left(1 - \frac{h}{a}\right)$$

$$= -\frac{1}{a}\left(1 - \epsilon \sin^2\phi\right)\frac{dN}{d\lambda}\left(1 - \frac{h}{a}\right)$$

$$= \left[1 - \frac{h}{a}\right]\sec\phi\left[\frac{1}{a}\sin\lambda\left(U\sin\phi'_0 + V\cos\phi'_0\right) - \cos\lambda\,\delta\xi_0\right]\operatorname{cosec} 1'' \textit{ seconds.}$$

As with latitude, the first term is unity when the station is near sea-level.

The change in deviation in the prime vertical $(\delta\xi) = -\delta\lambda \cos\phi$.

(*c*) In a horizontal side BC, $\delta(\text{scale}) = \frac{B''C''}{B'C'} - 1 = \frac{N}{a}$, N being the height of S_2 above S_1. If $\delta(\text{scale})$ is expressed in units of the 7th decimal of the log and N in feet, $\delta(\text{scale}) = 0{\cdot}207\ N$.

If B and C are of unequal heights an additional term is necessary to allow for the components in the line BC of the height terms in the formulæ for change of latitude and longitude at B and C given above. $\delta(\text{scale})$ then becomes :—

$$N/a + (h_1 - h_2)\ (\delta\eta \cos\beta + \delta\xi \sin\beta) \sin 1''/l$$

or $0{\cdot}207\ [N + a\ (h_1 - h_2)\ (\delta\eta \cos\beta + \delta\xi \sin\beta)\ \sin 1''/l]$ in the 7th decimal of the log,

where h_1 is the height of B, h_2 the height of C, and β the azimuth of C at B (i.e., the azimuth from h_1 towards h_2). $\delta\eta$ and $\delta\xi$ are in seconds, and N_1, h_1, h_2, a and l in feet. β is measured from south.

(*d*) In a horizontal side δ (azimuth) = azimuth of B″C″ *minus* azimuth of B′C′ (measured clockwise from south) = $\delta\lambda \sin\phi$†, $\delta\lambda$ being given by (*b*) above.

If B and C are of unequal heights there is an additional term to allow for the components at right angles to BC of the height terms in $\delta\phi$ and $\delta\lambda$. δ (azimuth) in seconds is then :—

$$\delta\lambda \sin\phi + (h_1 - h_2)\ (-\delta\eta \sin\beta + \delta\xi \cos\beta)/l$$

where l is the length of BC in the same units as h_1 and h_2. $\delta\lambda$, $\delta\eta$ and $\delta\xi$ are in seconds.

24. Application of the formulae.—The above formulæ show that to convert latitude, longitude, distance and azimuth from one spheroid to another, it is necessary firstly to apply corrections which depend only on the latitude and longitude of the point considered, and secondly further corrections depending upon height. The first corrections are generally the largest and

* There is a possibility of confusion here. On page 40 λ has been defined as longitude east of the origin, and the expression for the change of longitude as thus defined will be $\delta\lambda$ as given by the formula *minus* $\delta\lambda_0$, its value at the origin. In practice, however, longitude is always quoted as east of Greenwich, which is not geodetically connected to the Indian survey and whose longitude does not share changes arising from changes in the Indian spheroid. The expression now given for $\delta\lambda$ is therefore applicable to the Greenwich longitude.

† This is immediately derivable from the fact that the deviation in the prime vertical ξ equals both $(A - G) \cot\phi$ where A and G are astronomical and geodetic azimuths, and also $(A - G)\cos\phi$ where A and G refer to longitude. It follows that $\delta G \cot\phi = \delta G \cos\phi$.

vary slowly from place to place. For conversion from Everest's spheroid to a spheroid with International axes,* the necessary corrections are shown in Plates XI and XII. Plate XI gives $\delta\eta$ or $-\ \delta\phi$ and $\delta\xi$ or $-\ \delta\lambda \cos\phi$†. Plate XII gives δ (azimuth) and δ (scale) in units of the 7th decimal of the log. N‡ is obtained in feet by multiplying δ (scale) by 4·81. These charts as they stand are sufficient for converting scale, azimuth, and deviation of the vertical, but more detailed charts or calculations will be necessary for the conversion of latitudes and longitudes for ordinary survey purposes.

Distances on Everest's spheroid are at present expressed in Indian feet (see Appendices X and XI), and the axes of the spheroid used are automatically in the same units. After conversion to the International spheroid it will be more convenient to adopt British feet (of 1926) both for log sides and for the axes. All log sides will therefore require a correction of $-$ 0·000 0015 in addition to the corrections of Plate XII. See Appendix XI, para 5, footnote.

The corrections depending upon height (if required : but see para 25) must be separately calculated for each station and side where they are not negligible, but they are generally very small. Thus the correction to latitude $h\,\delta\phi/a$ equals $0''\cdot005$ when h is 5000 feet and $\delta\phi$ is $20''$. The latter is a larger value than ever occurs (see Plate XI), and since latitudes are generally only quoted to the nearest $0''\cdot01$, it will almost invariably be possible to neglect the height term. The same applies to longitudes.

The correction to scale is less frequently negligible. It amounts to less than one in the 7th decimal of the log if $(h_1-h_2)\ \theta/l$ is less than 250, where h_1-h_2 is measured in feet, l in miles, and θ seconds is the component of the deviation along the side. Now θ/l will very rarely exceed unity, so that this term will always be negligible if the difference of height in a side is less than 250 feet. In a very large part of India θ will always be less than $5''$, and the limit can be raised to 1000 feet except in short sides. Further there is really no need to be strict about limiting errors to 1 in the 7th decimal. This is only 1 in 4 million, so isolated cases may be ignored if they exceed the limit even 5 or 6 times, or more in a side which is exceptionally short.

Azimuths are recorded to the tenth of a second, and the height corrections to them are negligible under the same conditions as have been given above for scale.

As explained in Appendix XI a peculiar inconsistency has caused all published Indian longitudes on Everest's spheroid to be too large by $3''\cdot16$. Allowance is made for this when applying Laplace's equation and when deducing the deviation of the vertical from longitude observations. Charts exhibiting the form of the geoid relative to Everest's spheroid are thus free from this error, and deduced values of ξ_0 on the International spheroid are also correct. A constant error in longitude is not very inconvenient, but it will be desirable to remove it when changing spheroid, so that a term $-3''\cdot16$ should be added to the formula of para 23 (b). No correction on this account has been included in Plate XI, because it gives the changes in ξ, which has been correctly computed as stated above.

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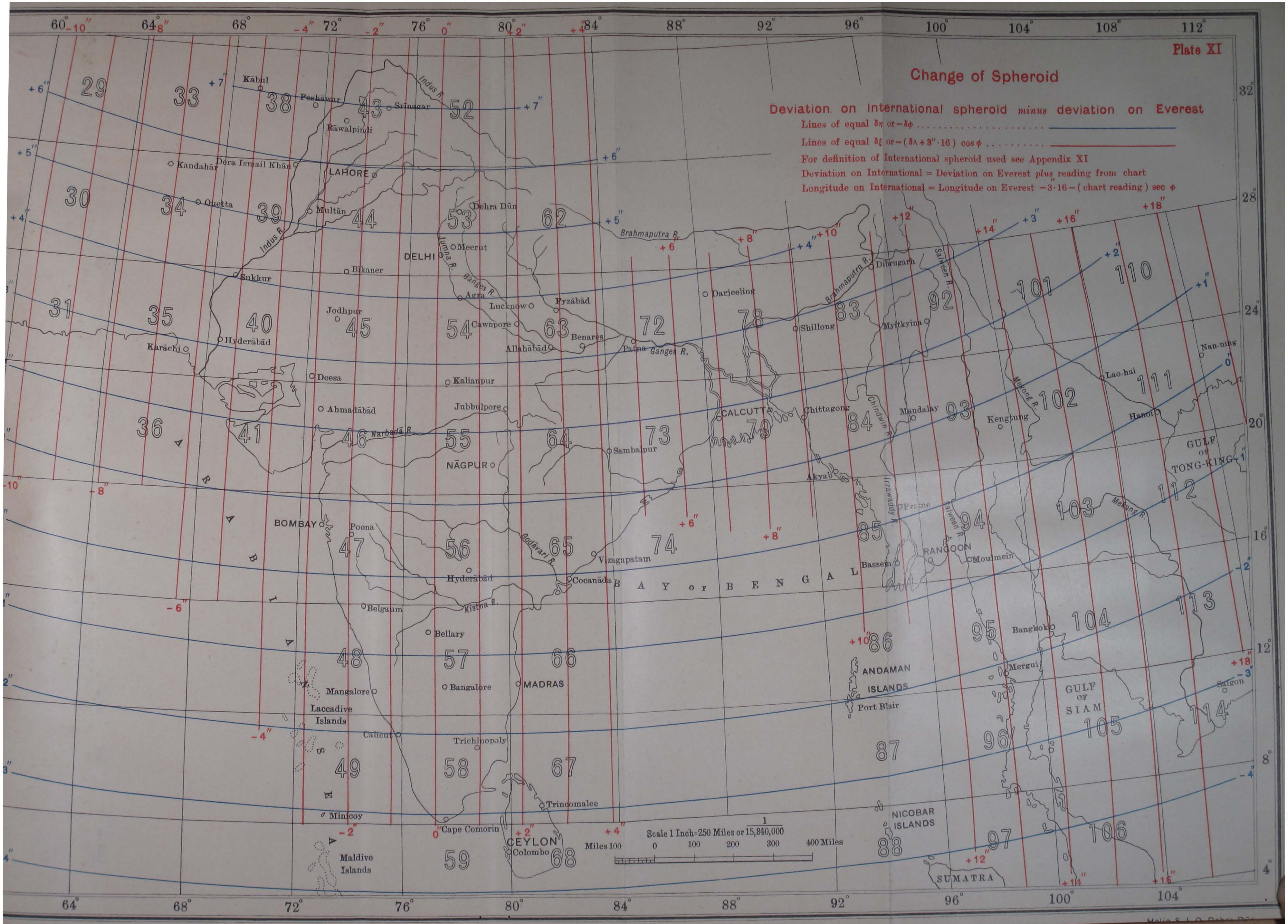
* With deflections at the origin securing the best possible fit to the geoid as known in 1927. See Appendix XI, para 5.

† Or more correctly $-(\delta\lambda + 3''\cdot16)\cos\phi$. See the last sub-para of para 24.

‡ Mr. B. L. Gulatee has given a chart of N in Geodetic Report 1934, Chart XXVIII.



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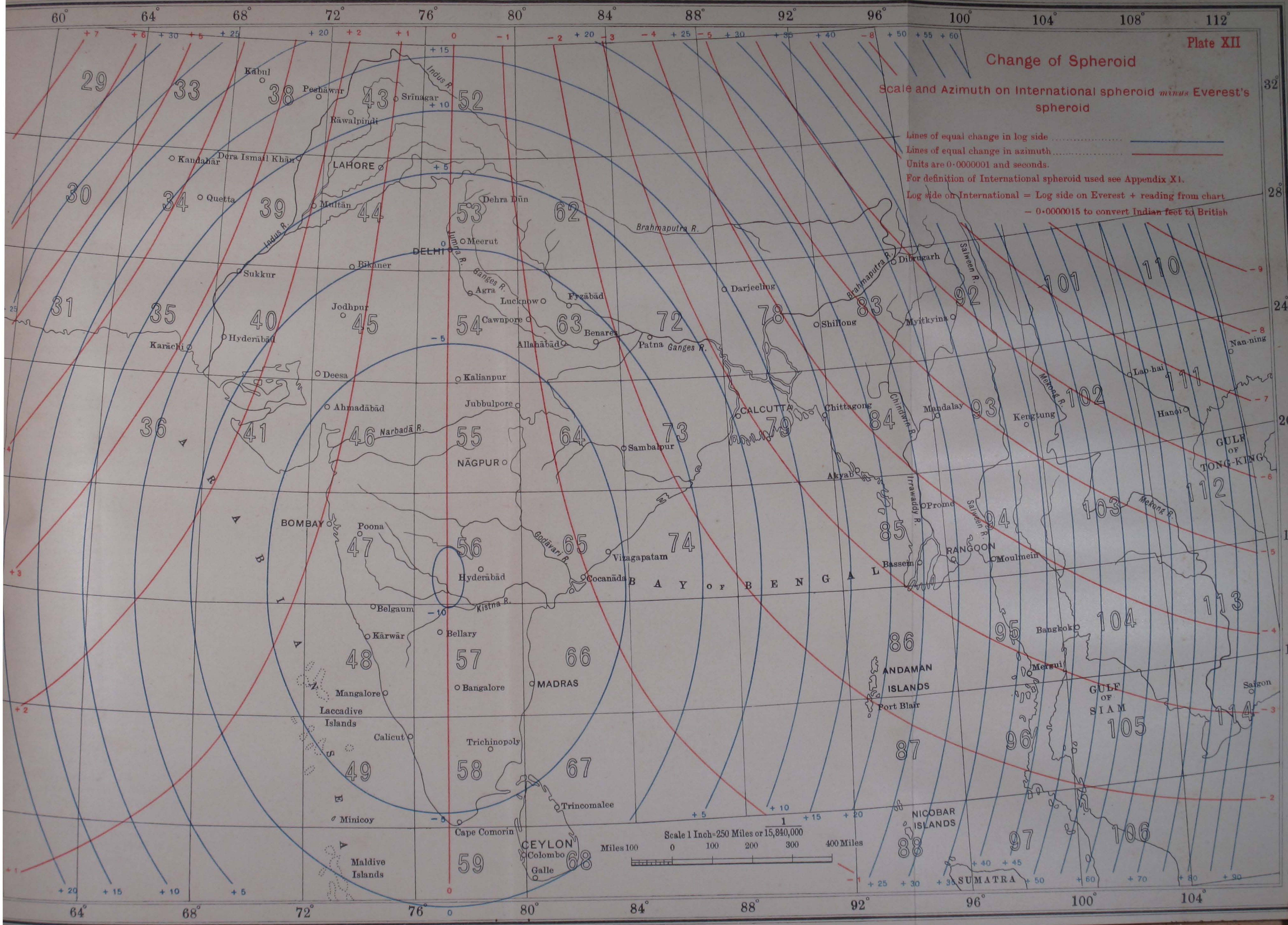

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25. Neglect of deviation corrections.—As stated above, the treatment proposed in para 23 is correct provided the computations on the first spheroid are correct. In India, as now readjusted, the base-lines and Laplace stations are correctly computed, but the deviation corrections have in general been ignored. It happens that Everest's spheroid is not at all a close approximation to the geoid, and there are areas in which it is inclined at 10″ or even 15″ to a spheroid, such as the International, which fits the geoid well. In these areas, especially, it is necessary to examine the effect of having computed on Everest's spheroid without the corrections, and to be satisfied that the application of paras 23 and 24 will secure the same results as would have been arrived at if the triangulation had been fully recomputed (without the deviation corrections) on the new spheroid.

The deviation of the vertical at any point on Everest's spheroid may be considered to consist of two parts, one θ_1 due to the general bad fit of the spheroid, and the other θ_2 due to irregularities of topography and crustal density. This separation is of course arbitrary, but θ_1 may be defined as the inclination between Everest's spheroid and any well-fitting spheroid such as the International, while θ_2 is the deviation remaining on the better spheroid. Since the effects of each are very small, they may clearly be considered separately, and their results superimposed. The first part, θ_1, varies slowly from place to place, the effect of ignoring it in the original computations would be remedied by recomputing on a better spheroid, and it is the only part of the deviation which has to be considered in connection with change of spheroid. The second part, θ_2, often varies sharply between adjacent stations, it is independent of the spheroid originally used for computation, and its neglect results in random errors which can only be remedied by observing the deviation at all stations where its effects are not clearly negligible.

Suppose now that a triangulation series lying between two (horizontal) base-lines is computed on a good spheroid S_2 for which $\theta_1 = \text{zero}$. As stated above, θ_2 can be considered entirely separately. Then, apart from the effects of θ_2, if all observations were perfect, the triangulation starting from one base would exactly reproduce the reduced measured length of the second, for there would be errors of neither observation nor computation. Now let the same piece of triangulation be recomputed on another spheroid S_1 (e.g., Everest's) which lies below S_2 by distances N_1 and N_2 at the two base-lines. On S_1 the reduced length of the opening base will be decreased in the proportion $1 : (1 + N_1/a)$, and if θ_1 (now no longer zero) were ignored, the triangulated value of the closing base and all other horizontal sides would be decreased in the same proportion, since the solutions of the triangles would be absolutely identical* apart from change in the opening side. The error in the triangulated length at the closing base-line would then be $(N_2 - N_1)/a$, and a similar expression would give the error of any other horizontal side. In a side which was not horizontal there would also be a random error depending on the difference of height at its two ends, since the conversion from one spheroid to the other involves a height term. (See para 23). It is clear therefore that one effect of neglecting the deviations θ_1 is to incur a steady accumulation of scale error (between horizontal base-lines), which is equal to $N_2/a - N_1/a$, where N_2/a and N_1/a are the changes of scale at the two base-lines resulting from conversion from the bad spheroid to a good one.

* There would be a minute change in the spherical excess, but that would be distributed among the angles of each triangle in the same way as errors of observation. If angles were equally weighted, its effect would be zero, and in any case certainly negligible.

Now Everest's spheroid is an extreme example of a bad spheroid, but between adjacent base-lines the difference of separation between it and the International never exceeds 108 feet. It averages 40 feet. These figures correspond to 22 and 8 in the 7th figure of the log, which are considerably smaller than the actual misclosures shown in Chapter II, Table 3 (Maximum 166, average 47). If the neglect of deviation corrections was the sole source of error in triangulation, the scale misclosure resulting from it would be distributed evenly* between base-lines without considering the strength of the intermediate triangulation. Actually the varying strength of intermediate series has been taken into account when distributing base-line misclosures, but as sources of error other than the deviation are responsible for the greater part of the misclosures, this course has been correct, and the inaccuracy resulting from the uneven distribution of the small deviation correction error is small compared with the inevitable uncertainty in the distribution of ordinary errors of observation. If this had not been the case, it would have been easy (in Chapter I) to have distributed the error arising from Everest's spheroid first, in the way best suited to it, and then to have distributed the remaining closing error according to the strengths of the series.

The effect on azimuth may be seen in a similar way. Suppose the series is first computed on an ideal spheroid S_2 so that the azimuth closes exactly on a terminal Laplace station. Then let it be recomputed on an imperfect spheroid S_1. The azimuth at the initial Laplace station will be changed by $-\delta\lambda_1 \sin\phi_1$ and at the terminal station it will be changed by $-\delta\lambda_2 \sin\phi_2$. There will therefore be an azimuth closing error of $\delta\lambda_2 \sin\phi_2 - \delta\lambda_1 \sin\phi_1$ similar to the scale error of $(N_2 - N_1)/a$ mentioned above. In the case of azimuth however, the computations on the two spheroids will not be identical, since the reverse azimuth in each side differs from the forward azimuth by $180^\circ + \Delta\lambda \sin\phi$, and the quantity $\Delta\lambda \sin\phi$ depends on the spheroid used. The total of the changes in $\Delta\lambda$ between the two Laplace stations is $\delta\lambda_1 - \delta\lambda_2$, and the change in ϕ is $+\delta\eta$, so the total change in all the terms $\Delta\lambda \sin\phi$ is $(\delta\lambda_1 - \delta\lambda_2)\sin\phi_m + \delta\eta_m \Delta\lambda \cos\phi_m$, where ϕ_m and $\delta\eta_m$ are appropriate mean values of ϕ and $\delta\eta$, and $\Delta\lambda$ is the extent of the series in longitude. Then the total closing error is

$$(\delta\lambda_2 \sin\phi_2 - \delta\lambda_1 \sin\phi_1) + (\delta\lambda_1 - \delta\lambda_2)\sin\phi_m + \delta\eta_m \Delta\lambda \cos\phi_m.$$

Like the scale error $(N_2 - N_1)/a$, this quantity varies fairly regularly along a series, and is much smaller than the closing errors due to errors of observation. It also is therefore adequately dealt with by ordinary distribution of error.

It has thus been shown† that neglect of the deviation corrections arising from the use of a bad spheroid introduces into the computations on that spheroid :—

(1) A cumulative error in scale.

(2) A cumulative error in azimuth.

(3) Small, random, non-cumulative errors in scale, azimuth and position, depending on the height of each station.

The cumulative errors of scale and azimuth are normally less than those arising from errors of observation, and are adequately dealt with by the ordinary dispersal of error between base-lines and Laplace stations, while the

* Not quite evenly, since series are not quite straight and θ_1 is not quite constant between base-lines, but the word "evenly" is approximately correct.

† Concrete instances are sometimes clearer than a general argument. Appendix XIII deals with a concrete case by actual application of the deviation corrections.

random errors depending on height are exactly equal to the correction which would be applied when converting from the bad spheroid to a good one. **It is therefore concluded that a triangulation system which is adjusted on reasonably frequent and correctly reduced base-lines and Laplace stations can be described as correctly computed throughout for the purpose of para 23 (except for small errors equal to the height corrections), in spite of deviation (θ_1) corrections having been neglected. The formulæ of para 23, without the height corrections*, may therefore be used for its conversion to a new spheroid.** This statement is not invalidated by the possibility that random deviations (θ_2) may not be negligible, since their effects are the same on both spheroids. The change of spheroid can be correctly carried out irrespective of the accuracy of the triangulation on either spheroid.

Plate XI gives values of $\delta\eta$ and $\delta\xi$ which are similar to those given in the Supplement to Geodetic Report Vol. VI Charts XIS and XIIS, which were based on Dr. J. de Graaff Hunter's formulæ for integration along geodesics. The greatest difference is $0''\cdot5$, or 50 feet. This is a small amount, not many times greater than the probable error of position resulting from errors of triangulation, and less than the changes arising from the new adjustment, so that the necessity for a new method is not very pressing. It is thought that the chief advantage of the method now used lies in the simplicity of the principles underlying it, which perhaps make it more suitable for general use.

* i.e., with h, h_1 and h_2 placed equal to zero. In the few places, such as the Dehra Dūn and Kêng Tung base-line extensions, where the deviation corrections have been applied, this is not strictly correct. Strictly, if height corrections are going to be ignored when converting to the International spheroid, these base-line extensions should have been computed on Everest's spheroid with the deviation corrections derived from deviations relative to the International spheroid (i.e., with corrections for θ_2 only), but the inconsistency will have negligible effects.

Chapter IV

CONCLUSIONS

26. Future field work.—Chapter II, Plate IX, gives the probable errors of position generated in the readjusted primary triangulation, after revision of the weak link east of Calcutta. It shows (see para 20) that the length and breadth of the country have been measured with a probable error of about 1 in 500,000. This is a high standard of accuracy, more than ample for all topographical purposes, while for scientific purposes it can certainly be described as adequate.

From the topographical and cadastral point of view a more important aspect is the probable relative error of comparatively near points, and in this respect the weakest parts of the primary triangulation are the three long parallel series known as the Gurhāgarh series, Rahūn series and Great Arc (section 24° to 30°). Plate X shows that the probable relative errors between the centres of these series are 14 and 11 feet respectively in intervals of 50 and 40 miles. A probable error of 14 feet is of no consequence for mapping at the scale of 1 inch to the mile, but if continuous mapping were ever undertaken on a much larger scale, such as the English 6-inch map, it would be desirable to tie the centres of these series together (Item 5 below). Apart from this weak point, it may be said that the accuracy of the primary triangulation makes it a sufficiently accurate basis for topographical or cadastral mapping on any scale whatever.

Plate X gives the probable error of the secondary triangulation relative to the primary, and shows that except in Kashmīr and the Pāmirs, it has generated no errors plottable at the scale of 1 inch to the mile. The next weakest parts are the long parallel chains crossing the western end of the Gangetic plain, similar to the primary series mentioned above, with probable relative errors between their centres of up to 15 feet in distances of 30 or 40 miles. If continuous large scale mapping were required, it would be necessary to tie together the centres of at least the Great Arc (24°–30°) and the Budhon, Rangīr and Amūa series (Item 6 below). There is no early prospect of such maps being required, but it is unfortunate that this weak part of the triangulation should lie in the area which will probably be almost the first to require them.

As regards the extent of the primary triangulation, there are a few places in which it does not extend up to the frontiers of India and Burma, and these gaps must be filled (Items 4, 9 and 11 below). There are two large areas, the North-east and South-west Quadrilaterals, which are devoid of primary triangulation, but their secondary triangulation is well controlled and has not generated serious errors of position except as mentioned above. At any rate until continuous 6-inch maps are required, there is no need to consider extending primary triangulation over these areas. There are a few places where series which are clearly part of the primary framework have locally fallen below the limits of accuracy prescribed for primary triangulation (viz:—The Gurhāgarh series in latitude 24½° to 26½°, the Bombay longitudinal series near Poona, and the Burma Coast Series between latitudes 14½°

and 16°). The reobservation of these series is not, however, advocated, since they have introduced little weakness. When the strengthening of an old series by new triangulation is under consideration, it must be remembered that the desired end will not be achieved unless undisturbed mark-stones can be relied on, or unless the new work contains its own base-line and Laplace station.

Between 1909 and 1917 a number of secondary series were observed which began to fill the gaps between primary series. This programme was then discontinued. The amount of secondary triangulation already observed is generally being found sufficient for the 1-inch map which is at present being surveyed, and no programme of secondary triangulation is now in hand. Where the interval between primary series is large, the topographical triangulation crossing the intervals is observed with some extra care. When larger scale maps are eventually required, a very extensive programme of secondary triangulation will probably be necessary.

The following paragraphs give in detail the work required to complete the primary triangulation of India and Burma. The items are given roughly in the order in which they are most likely to be undertaken. Items (1) and (4) to (11) are shown in the Frontispiece.

(1). East Calcutta primary traverse, to replace the weak East Calcutta longitudinal series, and to connect the primary triangulation of India to that of Assam and Burma. (Two seasons' work).

(2). In connection with item (1) base-lines will be observed at the south ends of the Brahmaputra and Eastern Frontier series.

(3). One or two Laplace stations are needed in the north of India near Peshāwar. They are in the programme for 1939–40.

(4). The Makrān series, running westwards from the west end of the present Makrān series, to the Irān frontier. (One season's work).

(5). Triangulation connecting the Gurhāgarh and Rahūn series and the Great Arc in latitude 27½°. (One season's work).

(6). Primary traverse connecting the Great Arc, Budhon, Rangīr and Amūa series in latitudes 26° to 28°. (Two seasons' work).

(7). Primary traverse replacing the Calcutta meridional secondary series. Connection between India and Burma is provided by item (1), but it will be unsupported, and the triangulation of Bengal and Assam will remain weak until this item is also completed. (Two or three seasons' work).

(8). A connection with the Siamese triangulation in latitude 16°. The Burma Coast series lies closely parallel to the Siamese triangulation, and it is desirable to tie them together at this point. (One season's work).

(9). Extension of the Great Arc northwards from Dehra Dūn to join the Kashmīr Principal series. Physical difficulties are great, and may be increased by the necessity for avoiding Tibetan territory, but it is very desirable to extend the Great Arc to the northern frontier of India, and the Kashmir series needs support at its eastern end. (Two or three seasons' work).

(10). Junction with French Indo-China. If the triangulation of French Indo-China is joined to that of Siam in latitude 20°, nothing will be necessary in Burma. On the other hand, it may be necessary to observe a series eastwards from Kēng Tung in latitude 21°. (One season's work).

(11). Chinese frontier and Sadiya series. Triangulation is required in the extreme north of Burma where it at present lies 150 miles inside the frontier. (Two seasons' work).

(12). Connection with any future primary triangulation in Afghānistān should be with the primary triangulation of India, and not with the secondary Peshāwar, Kurram and North Wazīristān series. (One season's work).

(13). Indo-European connection. For scientific purposes, this is the work most urgently called for*, but there are obvious political and financial difficulties in the way of the Survey of India working beyond the frontiers of India. There are four possible routes:—

(*a*) To Russia via Gilgit, following the route of the 1912-13 secondary triangulation. Little is known about the present state of primary triangulation in Russia, but it is possible that great progress is being made, and that a strengthening of the 1912–13 triangulation over the Pāmirs might provide a geodetic connection†. Improving the Indian section, if not actually impossible, will certainly be a very difficult matter.

(*b*) To Russia via western Afghānistān. There are probably no physical difficulties, but political and financial difficulties in the way of the Survey of India contributing to the work would probably be insuperable, while Afghānistān may not undertake geodetic triangulation itself for very many years.

(*c*) From Baluchistān through south and west Irān and Asia Minor. The same difficulties arise as in (*b*).

(*d*) For a short distance along the south coast of Irān, across the Gulf of Omān, along the south coast of Arabia, and so joining with the African 30th meridian arc via future Italian triangulation in Abyssinia. The Arabian section may present serious difficulties, but since the country is politically undeveloped, it might be easier for the Survey of India to work on this route than on either (*b*) or (*c*).

27. **The preservation of the triangulation.**—Primary triangulation stations in India are marked by a circle and dot cut on rock or more usually a loose stone. Above the mark is built a low pillar of stone or bricks, and the whole is surrounded by a large platform of loose stones and covered by a cairn, or in flat country a high brick tower may have been built over the mark. In jungle areas the station is liable to be destroyed by animals or vegetation: tower stations are destroyed by rain: many stations have been dug up by treasure-seekers, especially in Burma: stations in northern India and Burma are liable to disturbance by earthquake: and stations built on alluvium are liable to subsidence and possibly to movement arising from creep of the alluvium. The result is that, whereas the great majority of the stations can be identified within one or two feet, a rapid decrease is taking place in the number of pairs of stations which can give a value of scale and azimuth of geodetic accuracy. The triangulation is an adequate basis for topographical or cadastral work, and gives values of the latitude and longitude which are adequate for future geodetic extensions, but the time is fast coming when

* It must be remarked that full value will not be derived from connection with European surveys unless the triangulation is accompanied by a line of astronomical stations at sufficiently close intervals to provide a section of the geoid.

† So far as it was known in 1930, the state of the Russian triangulation in Turkistān is described in Geodetic Report Vol. VII, pages 6 and 7. At that time an early improvement of the connection did not seem probable.

future geodetic revisions or extensions must necessarily be provided with their own base-lines and Laplace stations*. This policy has in fact largely been followed in recent years, since modern triangulation series in Baluchistān, Assam and the Shan States of Burma are all provided with modern base-lines and Laplace stations.

With increasing development of the country it may be necessary for a field detachment to visit old stations, and replace their structures by monuments of a more modern type, but for the present it is thought that the best protection is provided by the inconspicuous pile of debris by which many are now covered. In addition, an annual report on their condition by the local authority impresses the villagers with the importance of the site, and with the necessity for preserving it as far as possible.

28. **Future adjustments.**—In India, excluding Assam and Burma, the corrections to geodetic positions on Everest's spheroid given in Plate VII should be final for all time, in so far as they are required for studies of deviations of the vertical. Any future changes are very unlikely to exceed $0''\cdot1$ of latitude or longitude. In Assam and Burma it is possible that revision of the East Calcutta series (item 1 of para 26) may introduce changes of $0''\cdot2$ or $0''\cdot3$, and a redrawing of the chart on this account may be necessary.

As a basis for topographical and cadastral maps, most of the figures given for India in Table 1A should also be final, but it may be desirable to make changes on account of:—

(*a*) Laplace stations near Peshāwar (Item 3 of para 26). They can reasonably be ignored if they suggest corrections of less than (say) $2''$ of azimuth.

(*b*) Future triangulation connecting the Gurhāgarh series, Rahūn series and Great Arc. It will almost certainly be possible to leave the ends of these three series unchanged, as in Table 1A, and so to limit changes to the three series themselves.

In Assam and northern Burma, a complete reconsideration of Table 1B will be necessary, taking into account:—

(*a*) The revision of the East Calcutta series.

(*b*) The connection between the 1934–36 Assam longitudinal series and older work near Gauhāti, completed in 1937–38.

(*c*) Laplace stations at Taungpila and Toungoo, north of Rangoon, observed in 1937–38.

(*d*) The revision of the Calcutta meridional series (Item 7 of para 26), and the new Chinese frontier and Sadiya series (Item 11), although it may not be convenient to wait until these are completed.

In southern Burma it will be necessary to undertake an adjustment in co-operation with the Survey departments of Siam and French Indo-China as soon as the geodetic triangulation of those countries reaches a reasonable state of finality. The positions of Moulmein and Kēng Tung and the Burmese triangulation between them may be held fixed, while the Burma Coast series south of Moulmein with the Siamese and Indo-Chinese triangulation may be adjusted on to these fixed series.

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* A secondary series bridging a gap between two primary series can, if necessary, obtain its scale and azimuth from a single fixed position at either end, instead of starting and closing on two sides of doubtful reliability. This deprives it of any check from its closing error in scale, azimuth and position, and so is to be avoided if possible.

29. Adoption of the new adjustment, and change of spheroid.— The new adjustment can be adopted for scientific purposes at once, but its adoption as the basis of topographical surveys is a very different matter. It will first be necessary to adjust all the primary triangulation series between the fixed positions given in Tables 1A and 1B (after revision), as described in para 5 and Appendix III, and the secondary series as described in Appendix IV. This is a practicable piece of work, although large, and the real objections to adopting the new adjustment lie in the necessity for republishing the 500 triangulation pamphlets in which the results are given, and in the necessity for applying corrections to the manuscript records of topographical triangulation whenever use is made of them*. These objections are so weighty that the new adjustment cannot possibly be brought into use until the adoption of the International spheroid introduces even larger changes, at which time it will be worth-while to introduce the new adjustment as well.

The triangulation of India and Burma, as computed on Everest's spheroid is self-consistent and accurate, but the use of this spheroid with axes 3,000 feet smaller than those of the earth does result in systematic differences between triangulated and astronomical positions in outlying areas. Such differences are not errors, but if they are sufficiently large, they are liable to be so described. Plate XI gives the probable values of these differences, and shows that they amount to 10″ and 14″ in longitude on the western and eastern frontiers respectively. The triangulation of Siam conforms to that of India within 1″ or 2″ of latitude and longitude†, and that of French Indo-China may also conform‡, but it is quite certain that surveys in Malaya and those eventually undertaken in Irān and China will not conform to India, and that discrepancies of about a quarter of a mile will arise on the frontiers. The objections to changing spheroid are even greater than those to adopting the new adjustment, for the changes are noticeable on a 1-inch map, and in addition to republishing all the triangulation pamphlets, it will be necessary to shift the detail of all maps with reference to their graticules. The latter not only involves much work, but for a period of 20 or more years imposes on the country two incomplete sets of inconsistent maps. The objections to a change are so great that Everest's spheroid is likely to be retained for a very long time, but it is thought that a change will eventually be inevitable, like the change of longitude adopted in 1905 many decades after the approximate error in the old longitude was known.

It is consequently thought that the introduction of the International spheroid, and with it the general use of the new adjustment, will be postponed until some distant date when (as in 1905) the current system of mapping is decided to be obsolete and in need of far-reaching changes. The future introduction of metre contours, which are also likely to be resisted for a long time, may provide an opportunity for introducing the new spheroid.

Although the axes of the International spheroid are unlikely to be changed, it will be premature to adopt the International spheroid until the deviations accepted at the origin can be placed on a more international basis than at present. As at present accepted they are derived from fitting the spheroid to the geoid in India and Burma, but there is no certainty that the spheroid so oriented would also fit the geoid in other more distant countries.

* Changes of under 50 feet hardly show on a one-inch map, but changes of 100 or 200 feet just do, so that the adoption of the new adjustment would also introduce some mapping difficulties.

† Geodetic Report, Volume VII, page 5.

‡ There is no direct connection at present.

A final determination of the deviation of the vertical at the origin can only be obtained by either :—

(*a*) A world-wide programme of gravity observations as proposed by Dr. J. de Graaff Hunter in Philosophical Transactions of the Royal Society, Volume 234 of 1935.

or

(*b*) A connection with Europe, and if possible Australia, by triangulation and geoid section.

As soon as the deviations at the origin are obtained by one of these means, it will be time to convert the adjusted values given in Tables 1A and 1B, after revision if necessary, to the final International spheroid. The primary and secondary triangulation can then be adjusted on to these positions as opportunity occurs, and kept in manuscript, so that it will be ready for use and publication when the time for its general adoption eventually comes.

APPENDIX I

REDUCTION OF BASE-LINES TO SPHEROID LEVEL

For computation on any particular spheroid a base-line must be reduced to "sea-level" by correction for its height above the spheroid used. In the original adjustment the 1830–70 base-lines were corrected for their spirit-levelled or geoidal heights, while annual reports on later base-lines have (in some cases) given figures corrected for only preliminary values of their heights above the spheroid.

Table 7 gives the heights of the base-lines above the geoid; the separations between the geoid, International spheroid*, and Everest's spheroid; the heights and lengths previously accepted; and those now accepted for computation on Everest's spheroid.

The height of the geoid above a spheroid can only be determined by an extensive series of latitude and longitude observations over the whole area of survey. This has been done in India and Burma, and the heights of the base-lines above Everest's spheroid are now correctly known to within 20 feet, which corresponds to a change of only 1:1,000,000 in the reduced length. In some Burma base-lines the heights now thought most probable differ by 8 to 10 feet from those which had been provisionally accepted, but no change has been made in the accepted reduced length, as the differences are not significant.

The separation between the geoid and the International spheroid has been taken from the latest geoid chart†, and that between the International and Everest's spheroids from Geodetic Report 1934, Chart XXVIII‡.

The lengths given are logs of the reduced lengths in Indian feet||. Heights are in feet.

Narrative descriptions of the measurement of the old base-lines are given in G.T. Vol. I. A report of the first measurement of the Mergui base is given in the General Report for 1881–82, and reports of the measurements of modern base-lines are given as follows:—Kēng Tung in Geodetic Report Vol. VII: Mergui, Amherst and Kalemyo in Geodetic Report 1933: Padag, Poona and Nāmtiāli in Geodetic Report 1934.

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* See Appendix XI for full definition of the International spheroid as used in India.
† Geodetic Report 1936, Chart VII.
‡ It can also be deduced from Plate XII of this volume.
|| See Appendix XI.

TABLE 7.—*Reduced lengths of Base-lines.*

Base-line	Dates	Height of: Base above Geoid*	Height of: Geoid above International	Height of: International above Everest	Old Figures: Accepted in	Old Figures: Height*	Old Figures: Log reduced length	Now accepted for Everest's spheroid: Height*	Now accepted for Everest's spheroid: Log reduced length
		feet	*feet*	*feet*		*feet*		*feet*	
ironj ...	1837-38	1529	+ 31	− 31	G.T. II p. 261	1529	4·584 4842	1529	4·584 4842
ehra Dūn ...	1834-35	1958	+ 25	+ 13	G.T. II p. 261	1958	4·593 1091	1996	4·593 1083
hach ...	1853-54	1015	0	+ 60	G.T. II p. 261	1015	4·616 4274	1075	4·616 4261
arāchi ...	1854-55	46	+ 30	+ 23	G.T. II p. 261	46	4·586 8609	99	4·586 8598
ider ...	1841-42	1975	+ 19	− 55	G.T. VI p. 33 & 18	1980	4·618 8693	1939	4·618 8702
alcutta ...	1831-32	10	+ 28	+ 27	G.T. VI p. 33 & 19	10	4·530 9666	65	4·530 9654
izagapatam ...	1862-63	311	+ 32	− 27	G.T. VI p. 33 & 20	311	4·541 3095	316	4·541 3094
angalore ...	1867-68	3110	+ 10	− 43	G.T. XII p. 58	3110	4·557 3103	3077	4·557 3110
ape Comorin † ...	1868-69	158	− 9	− 23	G.T. XII p. 58	158	4·605 4748	126	4·605 4755
onākhoda ...	1847-48	208	0	+ 40	G.T. VII p. 90	223	4·564 4979	248	4·564 4974
ēng Tung ...	1930-31	2550	+ 63	+210	G.R. 1934 p. 15	2815	4·584 9956	2823	The lengths given in the preceding column have been accepted, as the change of height is not significant.
mherst ...	1932-33	14	+ 91	+172	G.R. 1934 p. 15	279	4·648 9513	277	
alemyo North ‡	1932-33	473	+ 20	+110	G.R. 1934 p. 15	593	4·296 7040	603	
alemyo South ...	1932-33	422	+ 20	+110	G.R. 1934 p. 15	542	4·168 7325	552	
ergui ...	1932-33	10	+127	+210	G.R. 1934 p. 15	323	4·211 9202	347	4·211 9197
adag ...	1933-34	2680	+ 45	+ 73	G.R. 1934 p. 9	2817	4·712 1274	2798	4·712 1278
oona East ‡ ...	1933-34	1871	+ 15	− 43	G.R. 1934 p. 10	1826	4·353 5424	1843	4·353 5421
oona West ...	1933-34	1977	+ 15	− 43	G.R. 1934 p. 10	1932	4·088 8058	1949	4·088 8055
āmtiāli East ‡ ...	1933-34	324	+ 14	+135	G.R. 1934 p. 13	499	4·294 0886	473	4·294 0891
āmtiāli West ...	1933-34	325	+ 14	+135	G.R. 1934 p. 13	500	4·262 2572	474	4·262 2577

* In the ten 1830–70 base-lines this height is the height of one base-terminal, to the height of which measured lengths were first reduced. In the modern base-lines the height quoted is the mean height of e measured base-line itself.

† At Cape Comorin the length is that of the extended base-line.

‡ The Kalemyo, Poona and Nāmtiāli base-lines are each in two sections inclined at angles differing ppreciably from 180°.

Appendix II

LAPLACE STATIONS

A Laplace station is a trigonometrical station at which both azimuth and longitude have been astronomically observed. The longitude observation determines ξ, the east-west component of the deviation of the vertical, and so enables the astronomical azimuth to receive the correction $-\xi \tan \phi$ which arises from the instrument having been levelled parallel to the geoid instead of to the spheroid. See Chapter III, para 22. When the astronomical azimuth has been so corrected, it can be used to correct and control the geodetic azimuth brought up by the triangulation.

Before the introduction of wireless telegraphy a precise longitude observation was difficult and could only be undertaken in the immediate vicinity of a telegraph office. Longitude observations could not readily be made on a trigonometrical hill station. To form a Laplace station it was therefore necessary to carry the geodetic azimuth into the centre of a town, which in turn is generally difficult since it is essential to bring the azimuth to the Laplace station with fully the usual accuracy of primary triangulation. In consequence, until 1932 there existed in India only 5 primary stations at which astronomical azimuth and longitude observations were coincident.

Although coincidence of the two stations is desirable, it is not absolutely essential. The Laplace correction depends on the assumption that ξ is the same at both stations, with such exactitude that the difference in $\xi \tan \phi$ is negligible as an error in the deduced geodetic azimuth. In India the intervals between Laplace stations are often large, and the error generated in the triangulation between adjacent Laplace stations often amounts to several seconds. A Laplace station will then be valuable if the doubt in $\xi \tan \phi$ amounts to less than $\frac{1}{2}''$ or even $1''$. The permissible doubt in the identity of ξ at azimuth and longitude stations is therefore $\frac{1}{2}'' \cot \phi$, or even $1'' \cot \phi$. In the latitudes of Indian Laplace stations $\cot \phi$ varies between 1·7 and 7·0. It will be noted that on or very near the equator the longitude observation becomes unnecessary.

In G.T. Vol. XVIII (1906), Appendix V, Sir Sidney Burrard gave the rules that the azimuth and longitude stations may if necessary be separated by 5 miles in flat country, 2 miles if hills are visible on the horizon, and that they should be identical in hilly country. He gave a table of 20 stations at which these rules were satisfied*. The rules appear to be reasonably safe, and stations included in Sir Sidney Burrard's list can be admitted as good Laplace stations†.

Since 1932 about 300 longitude stations have been observed in order to trace the form of the geoid. To secure this large out-turn the route followed has so far as possible been confined to passable roads, and time has not been

* Including Vizagapatam and Kudankulam at which these distances were exceeded. Additional observations are now available to check Vizagapatam, while the low latitude of Kudankulam makes it acceptable. The list is also given in Professional Paper 16, page 165.

† Except that at four of the stations, Quetta, Orejhār (Fyzābād), Jalpāiguri and Kyaunggyi (Prome) there is no geodetic azimuth of primary accuracy, so that these stations cannot contribute to the adjustment of the triangulation.

spent in visiting inaccessible hill stations lying near the route. Very few identical Laplace stations have been established, but longitude has been observed in the same neighbourhood as many azimuth stations, sometimes in flat country and sometimes in hilly. It has therefore been necessary to re-examine Sir Sidney Burrard's rules to see what use can be made of these observations, and it appears that the rules can be relaxed under several circumstances, as follows.—

(*a*). South of latitude 12° the factor $\cot\phi$ exceeds 4·7. The South-east Coast series (55–63–60 in Plate 1) has a closing error of 9″ in azimuth, and a Laplace correction which is correct to 1″ or even 2″ is valuable. The several azimuth stations contained in the series can therefore be employed as Laplace stations if ξ can be estimated within 5″ or even 10″. The country is not hilly, some longitude observations exist for comparison, and it is thought that ξ can be estimated within probably 5″. On these grounds Kutipārai, Kallapat, Pātharankota and Manĕgandi have been admitted as Laplace stations.

(*b*). Recent longitude observations have been made at intervals of 10 or 15 miles along generally straight lines to provide sections of the geoid. If such a line passes within (say) 10 miles of an azimuth station, and if the 3 or 4 longitude stations nearest to it give equal values of ξ within one or two seconds, it is clear that a mean value (or the nearest) may be applied to the azimuth station, unless any abrupt topographical feature suggests that ξ is likely to be different there. On these grounds Māndvi, Dhauleshvar, Alsunda, Nughallibĕtta, Aknāpur, Anandbās, Madhpur, Lakhinagar, Gangapur and Semu Tan have been admitted as Laplace stations.

(*c*). If the country is hilly, the effects of the topography can be eliminated by calculating the "Hayford deflections"*, and assuming that the "Hayford deflection anomaly" at the azimuth station is equal to that at the nearest longitude station. On these grounds Vizagapatam Base N. End, Sānjib, Dāmargīda and Kodangal have been admitted as Laplace stations.

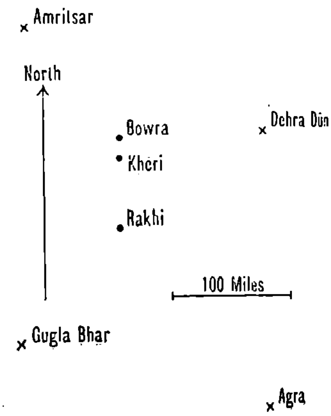
(*d*). Stokes' theorem shows that if the intensity of gravity is everywhere known the form of the geoid can be calculated, and hence the deviation of the vertical at any point. In Geodetic Report 1934, pages 138–141, Mr. B. L. Gulatee has given formulæ for calculating the Hayford deflection anomaly, given the Hayford gravity† anomalies. Method (*c*) assumes equality of the Hayford deflection anomalies at the azimuth and longitude stations, but even this assumption can be avoided if sufficient information is available

* The Hayford deflection is the deviation of the vertical calculated on the assumption that hills are of density 2·67, and that they are compensated by deficiencies of density on Hayfords' isostatic system. The Hayford deflection anomaly is the observed value of the deviation *minus* the calculated Hayford deflection. The utility of this method does not depend on the accuracy of Hayford's isostatic hypothesis, which is only used for convenience. For even if compensation is entirely absent the error in the assumed density distribution is deep-seated, and with hills of moderate size this will not cause the error of the Hayford anomaly to vary much in a distance of 10 miles. The important thing is to eliminate the effects of the local topography itself.

The words "deflection" and "deviation of the vertical" are synonymous.

† The Hayford gravity anomaly is the observed intensity of gravity *minus* that calculated on Hayford's system.

regarding the intensity of gravity. At present sufficient information is not available for the Hayford deflection anomaly to be calculable anywhere, but use has been made of the method as follows.—



During the adjustment of the triangulation there arose a pressing necessity for a Laplace station in about Lat. 30° Long. 76°, even if it might not be of the highest precision*. Near the required point there are three azimuth stations, Bowra, Kheri, and Rākhi, while the nearest longitude stations are Amritsar, Gūglā–Bhar, Agra and Dehra Dūn, 100 miles or more away. The Hayford gravity anomalies are fairly well-known within 140 miles (2°) of all these stations except Dehra Dūn †, and it is consequently possible to calculate the Hayford deflection anomaly at each except Dehra Dūn arising from gravity anomalies within 140 miles. Then knowing the actual Hayford deflection anomalies at Amritsar, Gūglā–Bhar and Agra, the difference between actual and calculated is the deflection anomaly due to anomalies of mass beyond 140 miles and to the general misfit of Everest's spheroid. Table 8 shows this difference to be $+0''\cdot 7$, $+2''\cdot 8$, and $+5''\cdot 5$ at the three longitude stations. Closer agreement would have been more satisfactory, although there is no reason for identity at the three stations. The mean $+3''\cdot 0$ may be accepted at the three azimuth stations with a probable error of $1''$, which introduces a probable error of $0''\cdot 6$ into the Laplace azimuth. Table 9 shows how the deviation at the three azimuth stations has been arrived at.

Table 10 gives results at all the Laplace stations which have finally been formed. It sometimes happens (e.g., at Dehra Dūn and Banog) that two or three Laplace stations lie close together, and it is better to mean the azimuth corrections derived from them than to accept each separately. In some such cases (e.g., Māndvi, Dhauleshvar and Alsunda) the similarity of the corrections deduced confirms the accuracy of non-identical stations whose results might otherwise have been considered untrustworthy.

Astronomical azimuths have sometimes been observed at two near stations (e.g., both ends of the Bangalore base-line), of which one is considerably closer to the longitude station than the other. In these circumstances only one has been entered in Table 10, but the fact that the two stations give similar values for the astronomical *minus* the geodetic azimuth provides a valuable check on the astronomical observation, and has been noted in the remarks column.

The last columns of Table 10 give the corrections which have to be applied to the triangulated azimuths as computed in Sironj-terms (see Chapter I, para 5), and as at present accepted and published, respectively.

* See Appendix III, para 8.

† Geodetic Report 1936, Chart XII.

In Table 10 some of the older astronomical azimuths have now been recomputed with modern values of the stars' declinations. The small correction for aberration which has sometimes been ignored, has now been included in all cases. At Dehra Dūn, Banog, Kēng Tung and Nāginimāra, where deviation corrections are large, the appropriate corrections have been applied. See Chapter III, para 22.

TABLE 8.—*Deviation in prime vertical at Longitude stations.*

(1) Longitude station	(2) Observed Deviation	(3) Hayford * deflection	(4) Hayford * anomaly (2) – (3)	(5) Hayford anomaly computed from gravity within 2°	(6) Effect of gravity beyond 2° (4) – (5)
	″	″	″	″	″
Amritsar ...	+ 5·3	– 1·9	+ 7·2	+ 6·5	+ 0·7
Gūgla-Bhar ...	+ 2·1	– 0·4	+ 2·5	– 0·3	+ 2·8
Agra ...	+ 7·8	+ 0·2	+ 7·6	+ 2·1	+ 5·5
				Mean ...	+ 3·0

TABLE 9.—*Deviation in prime vertical at Azimuth stations*

(1) Azimuth station	(2) Hayford anomaly computed from gravity within 2°	(3) Effect of gravity beyond 2° from Table 7	(4) Total Hayford anomaly * (2) + (3)	(5) Hayford* deflection	(6) Calculated Deviation (4) + (5)
	″	″	″	″	″
Bowra ...	+ 2·0	+ 3·0	+ 5·0	– 1·9	+ 3·1
Kheri ...	+ 2·6	+ 3·0	+ 5·6	– 1·5	+ 4·1
Rākhi ...	+ 2·0	+ 3·0	+ 5·0	– 0·6	+ 4·4

* See foot-note to page 55.

TABLE 10.—*Laplace Stations.*

Azimuth Stations									Longitude Stations						
Number on Plate I	Name	Latitude	Longitude	tan φ	Derivation of ξ *	ξ (Everest)	Date	Reference †	Name	Distance from Azimuth	Date	Reference †	Corrections to Sironj-terms Azimuth ‡	Mean	Correction to Published Azimuth	Remarks
		° ′	° ′			″				miles			″	″	″	
1	Kaliānpur ...	24 07	77 39	0·45	Close	+ 2·9	1836 & 1898	G. T. IV and Gen. R. 1898–99	Kaliānpur ...	0	1889–90	G. T. XV	0·0	0·0	0·0	
27	Dehra Dūn Obsy. ...	30 20	78 04	0·58	Close	−19·4	1853	G. T. IV	Dehra Dūn ...	1	1886–92	G. T. XV	− 3·1	−3·4	− 1·6	
27	Banog ...	30 29	78 01	0·59	Close	−22·4	1836	G. T. IV	Banog ...	0	1936	G. R. 1937	− 3·7			
28	Bowra ...	30 21	76 07	0·59	Method (d)	+ 3·1	1853	G. T. IV	...	...	...	...	+ 2·0	−0·1	− 1·3	Weak, but necessary.
28	Kheri ...	30 05	76 06	0·58	Method (d)	+ 4·1	1856	G. T. IV	...	...	...	...	− 2·3			
28	Rākhi ...	29 17	76 07	0·56	Method (d)	+ 4·4	1856	G. T. IV	...	...	...	...	+ 0·1			
19	Garinda ...	27 56	75 01	0·53	...	+ 2·1	1863	G. T. IV	Gūglā-Bhar ...	14	1935	G. R. 1935	+ 2·4	...	+ 1·3	Weak, but necessary.
5	Birona ...	24 26	72 13	0·45	...	− 0·4	1851	G. T. III	Deesa ...	13	1880–85	G. T. XV	− 0·1	...	− 1·1	Checked by Deesa Azimuth.
7	Chūtli ...	24 46	68 24	0·46	...	+ 7·2	1853	Manuscript	Kakeja ...	12	1928	G. R. IV	− 0·7	...	− 1·1	Weak.
Near 8	Kārothol ...	24 54	67 54	0·46	...	+ 7·3	1853	G. T. III	Sāhiji ...	19	1928	G. R. IV	− 2·6	...	− 2·9	Weak. Checks Karāchi.
8	Karāchi Obsy. ...	24 50	67 02	0·46	Close	+ 2·6	1855	G. T. III	Karāchi ...	2	1881–90	G. T. XV	− 2·4	...	− 2·6	Checked by Karāchi Base South.
18	Vijnot ...	28 02	69 51	0·53	Close	+ 9·6	1880	G. T. IV A	Vijnot ...	0	1934	G. R. 1935	− 0·6 R	...	− 1·5	
17	Yūsuf ...	27 51	68 26	0·53	...	+ 5·1	1858	G. T. III	Sūltān-kā-Got ...	18	1934	G. R. 1935	− 1·8	...	− 1·2	Weak.
13	Tozghi ...	28 53	62 15	0·55	Close	+12·5	1907	Gen. R. 1907–08	Tozghi ...	3	1934	G. R. 1935	− 0·2	...	+ 0·2	
35	Karaundi ...	23 11	80 00	0·43	Method (c)	− 7·1	1865	G. T. VI	Jubbulpore ...	3	1881–91	G. T. XV	− 0·8	...	− 1·1	
38	Tilabani ...	23 25	86 33	0·43	Close	− 5·0	1845	G. T. VI	Tilabani ...	0	1934	G. R. 1934	− 1·6	...	− 1·3	
38–39	Madhpur ...	23 10	87 45	0·43	Close	− 0·1	1868	G. T. VI	Madhpur ...	0	1934	G. R. 1934	− 1·9	...	− 1·3	

* "Close" indicates conformity with Sir Sidney Burrard's rules. "Method (a)" etc., refers to the four methods given in Appendix II. A blank indicates weakness. *(Contd.)*
† G. T. = G. T. S. Volume. Gen. R. = General Report. G. R. = Geodetic Report.
‡ When a Laplace Station is at or near a circuit point (see Plate I), the letters R or L indicate that the correction is to the right or left-hand branch respectively.

TABLE 10.—*Laplace Stations*—(contd.).

Azimuth Stations									Longitude Stations							
Number on Plate I	Name	Latitude	Longitude	tan φ	Derivation of ξ*	ξ (Everest)	Date	Reference †	Name	Distance from Azimuth	Date	Reference †	Corrections to Sironj-terms Azimuth ‡	Mean	Correction to Published Azimuth	Remarks
		° ′	° ′			″				miles			″	″	″	
						− 8·6			Sātten ...	13	1934	G. R. 1934	− 3·8			
38–39	Aknāpur ...	22 54	88 03	0·42	Method (b)	− 8·4	1869	G. T. VI	Dastānpur ...	10	1934	G. R. 1934	− 3 9	−4·4	− 3·9	Weak. Intermediate between Madhpur and Calcutta.
						− 4·5			Hākistāpur ...	13	1934	G. R. 1934	− 5·5			
39	Calcutta Base South	22 37	88 23	0·42	Close	− 7·2	1844	G. T. VI	Calcutta ...	5	1882–91	G. T. XV	− 6·6	...	− 5·9	
Near 66	Anandbās ...	23 21	88 23	0·43	Method (b)	− 6·5	1845	G. T. VIII	Simahāt ...	30	1934	G. R. 1934	− 5·5	−5·1	− 4·4	Weak. Checks Calcutta. Checked by Madhpur 23° 57′, 88° 29′.
						− 8·6			Sātten ...	27	1934	G. R. 1934	− 4·6			
43	Vizagapatam Base North	18 01	83 14	0·33	Method (c)	− 0·5	1863	G. T. VI	Waltair ...	21	1891–92	G. T. XV	− 5·3L			Weak.
43	Sānjib ...	17 31	82 41	0·32	Method (c)	− 2·2	1860	G. T. VI	Kottapalli ...	17	1936	G. R. 1936	− 4·7L	−5·2	− 1·0	
						+ 0·6			Kutturu ...	9	1936	G. R. 1936	− 5·6L			
						− 1·2			Ekeli Khurd ...	31	1935	G. R. 1936	− 1·6			
Near 47	Dāmargīda ...	18 03	77 40	0·33	Method (c)	− 2·0	1838	G. T. VI	Alīpur ...	26	1935	G. R. 1936	− 1·4	−1·3		
						− 3·0			Sadaseopet ...	35	1935	G. R. 1936	− 1·0			
						− 2·1			Ekeli Khurd ...	45	1935	G. R. 1936	− 4·0	−2·2	− 1·6	
Near 47	Kodangal ...	17 08	77 38	0·31	Method (c)	− 3·0	1872	G. T. XII	Alīpur ...	37	1935	G. R. 1936	− 3·7	−3·7		
						− 3·9			Sadaseopet ...	39	1935	G. R. 1936	− 3·4			
Near 47	Bolārum P.W.D. ...	17 30	78 31	0·32	Close	− 0·3	1904	Gen. R. 1903–04	Bolārum ...	0	1876–92	G. T. XV	− 1·6	−1·6	...	Weak Geodetic Azimuth.
Near 51	Colāba Obsy. ...	18 54	72 49	0·34	Close	+ 9·3	1839	Manuscript	Bombay ...	0	1876–92	G. T. XV	− 1·0L	...	− 1·9	

* "Close" indicates conformity with Sir Sidney Burrard's rules. "Method (a)" etc., refers to the four methods given in Appendix II. A blank indicates weakness. (*Contd.*)
† G.T. = G.T.S. Volume. Gen. R. = General Report. G.R. = Geodetic Report.
‡ When a Laplace Station is at or near a circuit point (see Plate I), the letters R or L indicate that the correction is to the right or left-hand branch respectively.

TABLE 10.—*Laplace Stations*—(contd.).

Azimuth Stations									Longitude Stations							
Number on Plate I	Name	Latitude	Longitude	tan φ	Derivation of ξ *	ξ (Everest)	Date	Reference †	Name	Distance from Azimuth	Date	Reference †	Corrections to Sironj-terms Azimuth ‡	Mean	Correction to Published Azimuth	Remarks
		° ′	° ′			″				miles			″	″	″	
51	Māndvi ...	18 38	73 32	0·34	Method (b)	− 2·0	1841	G. T. XII	Kanhe ...	9	1935	G. R. 1936	− 4·3			Weak. Hilly.
51	Dhauleshvar ...	18 26	74 10	0·33	Method (b)	+ 2·0	1838	G. T. XII	Koregaon ...	16	1935	G. R. 1936	− 3·4			
						+ 1·0			Kutbav ...	11	1935	G. R. 1936	− 3·0	−3·8	− 4·9	
51	Alsunda ...	18 27	75 01	0·33	Method (b)	+ 0·2	1863	G. T. XII	Kumbhargaon ...	19	1935	G. R. 1936	− 4·0			
						+ 1·2			Indapur ...	23	1935	G. R. 1936	− 4·3			
53	Mangalore ...	12 52	74 51	0·23	Close	+ 5·0	1873	G. T. XIII	Mangalore ...	0	1877–88	G. T. XV	− 1·9	...	− 3·9	
53–54	Nughallibĕtta ...	13 02	76 29	0·23	Method (b)	− 4·1	1871	G. T. XIII	Dandiganhalli ...	14	1935	G. R. 1936	− 4·5	−4·6	− 6·5	
						− 2·7			Rājapur ...	8	1935	G. R. 1936	− 4·8			
54	Bangalore Base SW.	13 01	77 35	0·23	Close	+ 0·3	1870	G. T. XII	Bangalore ...	0	1876–88	G. T. XV	− 7·0 L	...	− 5·3	Checked by Bangalore Base NE.
55	Anandalamalai ...	12 56	79 24	0·23	Method (b)	+ 2·9	1866	G. T. XIII	Chinna Samudram	2	1936	G. R. 1936	− 8·6 R	−7·4	− 4·8	Checked by Injambākam.
55	St. Thomas's Mount	13 00	80 12	0·23	Close	− 4·0	1880	G. T. XIII	Madras ...	6	1876–92	G. T. XV	− 6·3 R			
62	Kallapat ...	11 57	79 34	0·21	Method (a)	0	1879	G. T. XIII	...	...	...	...	− 8·5 L	...	− 4·9	
62–63	Nayinipiriyān ...	11 08	79 21	0·20	Method (a)	0	1879	G. T. XIII	...	...	...	...	− 7·1 L	...	− 2·2	Rejected. See Appendix III, para 9.
Near 63	Pātharankota ...	10 28	79 13	0·18	Method (a)	0	1877	G. T. XIII	...	...	...	...	−14·1 L	...	− 7·8	
63-64	Manĕgandi ...	9 46	78 55	0·17	Method (a)	+ 2·0	1876	G. T. XIII	...	...	...	...	−17·0 L	...	− 9·4	
64	Ramnad ...	9 22	78 49	0·16	Close	+ 1·4	1875	G. T. XIII	Ramnad ...	0	1933	G. R. 1934	−16·6	...	− 7·9	
60	Kutipārai ...	9 29	78 01	0·17	Method (a)	+ 4·0	1873	G. T. XII	...	...	...	...	− 8·4 R	...	− 8·2	
61	Kudankulam ...	8 10	77 41	0·14	Method (c)	+ 2·8	1869	G. T. XII	Nagarkoil ...	17	1888	G. T. XV	− 7·8	−7·5	− 7·2	
61	Rādhāpuram ...	8 17	77 42	0·15	Method (c)	+ 2·8	1869	G. T. XII	Nagarkoil ...	19	1888	G. T. XV	− 7·3			

* "Close" indicates conformity with Sir Sidney Burrard's rules. "Method (a)" etc., refers to the four methods given in Appendix II. A blank indicates weakness. (*Contd.*)
† G.T. = G.T.S. Volume. Gen. R. = General Report. G.R. = Geodetic Report.
‡ When a Laplace Station is at or near a circuit point (see Plate I), the letters R or L indicate that the correction is to the right or left-hand branch respectively.

TABLE 10.—*Laplace Stations*—(concld).

Number on Plate I	Azimuth Stations: Name	Latitude	Longitude	tan φ	Derivation of ξ*	ξ (Everest)	Date	Reference †	Longitude Stations: Name	Distance from Azimuth	Date	Reference †	Corrections to Sironj-terms Azimuth ‡	Mean	Correction to Published Azimuth	Remarks
67	Daulatpur ...	23° 09′	89° 43′	0·43	Method (b)	−11″·8	1868	G.T. VIII	Daulatpur ...	miles 0	1934	G.R. 1934	+ 1″·8	″ ...	+ 0″·3	
67–68	Gangapur ...	23 00	90 27	0·42	Method (b)	− 9·7	1866	G.T. VIII	Haripur ...	18	1934	G.R. 1934	− 0·5	−0·3	− 2·9	
						−10·8			Mālgaon ...	11	1934	G.R. 1934	0·0			
68	Lakhinagar ...	23 01	90 46	0·42	Method (b)	− 7·4	1866	G.T. VIII	Bāshakpur ...	10	1934	G.R. 1934	+ 5·2	+5·7	+ 1·8	
						− 9·7			Haripur ...	11	1934	G.R. 1934	+ 6·2			
69–87	Semu Tan ...	22 49	91 48	0·42	Method (b)	− 9·3	1865	Gen. R. 1864 65	Pokimura ...	15	1934	G.R. 1934	+ 3·1			
						−10·4			Maji Tan ...	9	1934	G.R. 1934	+ 3·6	+3·3 } +2·6	− 6·1	
						− 9·3			Betagee ...	15	1934	G.R. 1934	+ 3·1			
87	Nagarkhāna ...	22 23	91 49	0·41	Close	− 8·0	1905	Gen. R. 1904–05	Chittagong ...	3	1883–84	G.T. XV	+ 1·8	+1·8		
73–74	Tatalia ...	26 12	92 16	0·49	Close	− 2·7	1934	G.R. 1935	Tatalia ...	0	1936	G.R. 1936	+ 4·7	...	− 0·9	
75	Nāginimāra ...	26 49	94 50	0·51	Close	−27·5	1933 & 1935	G.R. 1935	Nāginimāra ...	0	1936	G.R. 1936	+ 3·3	...	− 7·2	
91	Dat Taung ...	20 13	93 01	0·37	...	− 7·4	1866	Gen. R. 1866–67	Akyab ...	9	1883–84	G.T. XV	+ 5·7 R	...	− 7·7	Weak.
90	Mingun ...	22 03	96 00	0·41	Method (b)	−19·7	1892	Gen. R. 1891-92	Paganyat ...	9	1932	G.R. 1933	+ 4·2 L	...	− 5·8	Checked by Sheinmaga.
86	Kēng Tung Base South	21 18	99 37	0·39	Close	−12·3	1931	G.R. VII	Kēng Tung ...	0	1933	G.R. 1933	+ 0·4 R	...	−12·1	
98	Taungzun ...	16 26	97 40	0·29	Close	−12·9	1884	Gen. R. 1883 84	Moulmein ...	5	1884	G.T. XV	− 2·0	...	−10·4	Checked by 5 near Azimuth Stations.
94–93	Taungpila ...	20 42	95 53	0·37	Close	− 7·9	1891	Gen. R. 1890–91	Taungpila ...	1	1937	G.R. 1938	+ 2·1	+1·5	− 9·4	Observed in 1937-38 too late for inclusion in the provisional adjustment of Burma.
94–93	Toungoo ...	18 56	96 26	0·34	Close	−15·1	1890	Gen. R. 1889-90	Toungoo ...	0	1937	G.R. 1938	+ 0·9			
101	Mergui West End ...	12 22	98 44	0·22	Close	−12·4	1882	Gen. R. 1881–82	Mergui ...	0	1937	G.R. 1938	− 1·1	−0·9	− 9·0	
101	Mergui East End ...	12 22	98 47	0·22	Close	−12·4	1882	Gen. R. 1881–82	Mergui ...	3	1937	G.R. 1938	− 0·7			

* "Close" indicates conformity with Sir Sidney Burrard's rules. "Method (a)" etc., refers to the four methods given in Appendix II. A blank indicates weakness.
† G.T. = G.T.S. Volume. Gen. R. = General Report. G.R. = Geodetic Report.
‡ When a Laplace Station is at or near a circuit point (see Plate I), the letters R or L indicate that the correction is to the right or left-hand branch respectively.

Appendix III

DETAILS OF THE 1937 ADJUSTMENT

(in amplification of Chapter I, para 5)

1. **Definitions.**—The meanings of the following expressions as used here and in Chapter I, para 5, are not self-evident.

(*a*) *Sironj terms.* Triangulation computed in Sironj terms has been computed from the Sironj base and adjacent Kaliānpur origin, along the routes shown in Plate I (each route terminates at an arrow head) ignoring other base-lines, Laplace stations, and circuit misclosures.

(*b*) *Circuit point.* A point marked by an arrow in Plate I, where two (or three) routes meet and a misclosure consequently occurs.

(*c*) *Junction point.* A point numbered on Plate I, including the circuit points. They have been placed at junctions or bends in the triangulation, or at Laplace stations.

(*d*) *Series.* As used in this Appendix and in Chapter I, para 5, the word series indicates the triangulation lying between two adjacent junction points. In its ordinary sense, as used elsewhere, it is any piece of triangulation which for one reason or another has been given an independent name.

(*e*) *Adjustment A.* The values of scale, azimuth, latitude and longitude arrived at by imposing on Sironj terms the corrections derived from all base-lines, Laplace stations and circuit misclosures of scale and azimuth, but ignoring circuit misclosures in latitude and longitude.

(*f*) *Traverse flank.* A continuous single line of sides of triangles proceeding from one end of a series to the other. In some methods of adjustment it is convenient to select such a line to typify the series, and these traverse stations are adjusted first. The remaining stations are then adjusted on to them.

(*g*) *The NW. Quadrilateral,* SW., SE., NE. Quadrilaterals and South Trigon are the five main divisions into which the Indian triangulation was divided for the purpose of the 1880 adjustment. Triangulation in Bengal and Assam, east of Calcutta, has now been removed from the NE. Quadrilateral, and treated together with the triangulation of Burma.

(*h*) *Published terms.* The data as published in the triangulation pamphlets, consisting of the "Final" adjusted data of the G.T. Volumes, the results of the 1916 Burma adjustment, and the provisional adjustment of other series which have been fitted into place from time to time.

(*i*) *Computation Volumes.* Manuscript volumes containing the original computations of a series, involving no adjustment other than figural (grinding of quadrilaterals etc.).

2. **Selection of circuits.**—The arrangement of circuits shown in Plate I arises from the desirability of obtaining the closing errors in Sironj terms

with the least labour. G.T. Volumes II, VI, VII, XII and XIV* give Sironj-terms scale, azimuth, latitude and longitude along selected traverse flanks of all the primary series dealt with in the old adjustment, and labour has been saved by again following the same routes. The original computations of later triangulation have generally been carried out in manuscript computation volumes in terms of the 1880 adjustment, so in these series it has now been necessary to obtain Sironj terms as described below in para 3.

It must be noted that the circuits formed in Plate I are not the separate small polygons into which the circuit lines divide the area. For instance, the polygon 24-23-18-10-11-16-24 is not a circuit. The circuit which closes at 24 consists of the two routes 3-19-20-21-22-23-24 and 3-4-5-6-7-8-9-10-11-16-24. In connection with triangulation the word "circuit" is in fact inappropriate, for although the closing error of a circuit in scale and azimuth is independent of the point chosen for the start and closure, that in latitude and longitude is not. Further, the usual property of circuits, that the closing error of a large circuit is equal to the sum of those of the smaller circuits into which it may be divided, does not hold. The so-called circuit misclosures are in fact the misclosures between computations along the two long routes by which each circuit closing point can be reached from the origin.

When selecting these circuits or routes from the origin, it is desirable to emanate from the origin by the strongest series, and to base the weaker series upon them. Thus, instead of basing series 20-21-22-23-24-25, 30-31 and 21-31-25 upon the weak series 3-19-20-30, it would have been better to have followed the routes 2-28-29-30-31-25-24, 31-21-22-23, 30-20-21 and 3-19-20. Apart from this case, the routes selected in the G. T. Volumes have been entirely convenient, and even in this case it has not been thought worth while to depart from them†.

The straight lines chosen to represent series should so far as possible lie within or close to the actual series, and it is obligatory that they should pass through any point where there is discontinuity‡ between the G.T. or computation volumes on which the adjustment is being based.

3. **Computation of Sironj-terms circuit errors.**—The closing errors of circuits which are identical with those of the old adjustment are obtainable from the G. T. Volumes almost directly. But when, as in most cases, the circuit contains any later series whose computations in Sironj terms are not already available, some fresh computation is necessary. It would be laborious to recompute all recent series entirely afresh in Sironj terms, and that can of course be avoided. It is only necessary to compute the changes $\delta_2 S$, $\delta_2 A$, $\delta_2 \phi$ and $\delta_2 \lambda$ at the terminal point of a series, which arise from small changes $\delta_1 S$, $\delta_1 A$, $\delta_1 \phi$ and $\delta_1 \lambda$ in scale, azimuth, latitude and longitude at its beginning. Generally the changes to be dealt within this and later stages do not exceed

* e.g., Vol. II pages 277–302. Vols. VI, VII, XII and XIV differ slightly from Sironj terms since they are based on the adjustment of the North-West Quadrilateral in Vol. II. The necessary modification is, however, easily made.

† Failure to follow the strongest route results in inconvenience later, not error. It does not result in the weaker series receiving undue weight in the final result, but only in corrections being larger than they need have been.

‡ By discontinuity is meant a change of terms, such as occurs at the beginning of a modern series which is based on the 1880 adjustment and so is not on Sironj terms: or such as at point 15 which in the computation volumes is reached by the two routes 9-15 and 12-15 with a discontinuity at 15. A simple correction to longitude (e.g., $-2' 27''\cdot 18$) does not constitute a change of terms.

150 in the 7th decimal of the log, 10″ in azimuth or 40 feet in ϕ or λ, and the average is very much less. With these small changes the following approximate formulæ hold with a sufficient degree of accuracy* :—

$$\delta_2 S = \delta_1 S$$

$$\delta_2 A = \delta_1 A$$

$$\delta_2 \phi = \delta_1 \phi + K_1 \delta_1 S + K_3 \delta_1 A$$

$$\delta_2 \lambda = \delta_1 \lambda + K_2 \delta_1 S + K_4 \delta_1 A \dagger$$

where $K_1 = -0\cdot122\, L \cos a,\ K_2 = -0\cdot122\, L \sin a$

$K_3 = +2\cdot56\, L \sin a,\ K_4 = -2\cdot56\, L \cos a$

$100\, L$ = length of series in miles

a = azimuth of the line on Plate I at its middle point. (From start to terminal, clockwise from south).

Units are 7th decimal of log for δS, seconds for δA and feet for $\delta\phi$ and $\delta\lambda$.

The values of K_1, K_2, K_3 and K_4 can be read immediately from a chart such as Plate I, by means of a suitable scale.

In criticism it may be said that the above formulæ introduce approximations (as in $\delta_2 A = \delta_1 A$) which are just large enough to affect the last significant figure (e.g., 0″·1 of azimuth) usually recorded. This is so, but the errors so arising are much smaller than the errors of observation and are really negligible. There is much to be said for keeping computations so simple that the essential features do not get lost in a maze of detail.

By means of these formulæ Sironj-terms values of the starting and terminal points of any series can be very easily obtained from its computation volumes, in whatever terms they may have been computed‡. To illustrate the process an example is given below :—

Example of the computation of Sironj-terms closing error at point 12.

Left-hand route 9-15-12.

The Makrān series 9–15 starts from Sūlimāni§–Andar of the Great Indus Series 8–25, and its computation volume accepts :—

Side	Azimuth	Latitude	Longitude	Reference
	° ′ ″	° ′ ″	° ′ ″	
5·209 0179	1 07 19·4	26 28 04·51	67 15 12·50	Vol. E 8/14

These are the "Final" adjusted terms of G.T. Vol. III, page 60*d*. It is not a traverse side

The nearest traverse side is Bhīt-Tikka, for which :—

	Side	Azimuth	Latitude	Longitude	Reference
Sironj terms	=5·047 2884	350 09 18·4	26 21 08·49	67 28 36·06	Vol. II. p. 289 & 300
Final terms	= 2946	18·4	08·65	36·06	Vol. III, p. 60*d*
∴ Sironj − Final	= −62	0·0	−0·16	+0·00	

These corrections have to be applied between 9 and 15, the terminal point of the Makrān series‖.

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* i.e., to 0″·01 of latitude or longitude, and 0″·2 of azimuth in any one series.

† In a few cases $\delta_1\phi$ has exceeded 1″, and a term $-\delta_1\phi\, 100\, (L/a) \tan\phi \sin a$ has been appreciable, and has been included in the expression for $\delta_2\lambda$. a = earth's radius.

‡ It is only necessary that the computations used should contain no circuit adjustments, and that the change of terms should not be unduly large.

§ The values of latitude and longitude refer to the first station mentioned (Sūlimāni), and the azimuth is that at Sūlimāni of Andar.

‖ The use of $\delta\phi$ and $\delta\lambda$ at the nearest traverse station, Bhīt, ignores the effect of the scale and azimuth discrepancies, −62 and 0·0, on the distance between Bhīt and Sūlimāni. This may sometimes amount to 0·01 in ϕ or λ, but not more. If a larger error was feared Sūlimāni could have been made a numbered junction point, and Bhīt–Sūlimāni treated as an independent series like 50–57, 70–77 or 79–80.

$\delta\phi = -0''\cdot 16 - 62\,K_1 + 0\cdot 0\,K_3 = -0''\cdot 16$

$\delta L = \;\; 0''\cdot 00 - 62\,K_2 + 0\cdot 0\,K_4 = +0''\cdot 08$

Verify continuity between computation volumes E 8/14 and E 8/23.

At 15 Buzgalaband-Kapar is the junction side with the Dālbandin series 15-12.

Makrān values	=5·082 6793	68 28 03·4	26 30 04·33	65 39 55·11	Vol. E 8/23
∴ Sironj terms =	6731	03·4	04·18	55·19	
Dālbandin terms* =	6793	04·7	04·23	65 37 28·01	Vol. E 16/25
				+ 2 27·18†	
				39 55·19	
∴ Sironj-Dālbandin =	−62	−1·3	−0·05	+ 0·00	

These corrections have to be applied between 15 and 12 the terminal point of the Dālbandin series.

$\delta\phi = -0''\cdot 05 - 62\,K_1 - 1\cdot 3\,K_3 = -0''\cdot 20$

$\delta L = +0''\cdot 00 - 62\,K_2 - 1\cdot 3\,K_4 = -0''\cdot 04$

At 12, the junction side with 11-12 is Pulchotau-Kisanen Chappar, for which

Dālbandin terms	=5·370 9007	79 27 33·3	29 11 06·86	65 05 47·77	Vol. E 16/25
				2 27·18	
				08 14·95	
∴ Sironj terms (L.H.) =	8945	32·0	06·66	14·91	

Right-hand route 9-10-11-12.

In exactly the same way 10-11 is based on the Great Indus Series, and the necessary changes are followed through to 12, for which are found:—

Sironj terms (R.H.)	=5·370 9001	79 27 33·7	29 11 06·64	65 08 14·99
Then closing error R − L =	+56	+1·7	−0·02	+0·08
			= −2	and + 7 feet

4. **Adjustment A.**—Plates II and III show how the Sironj-terms scale and azimuth misclosures have been distributed. The method by which the distribution is carried out calls for little explanation. The known errors at base-lines and Laplace stations are first provisionally distributed along the (generally strong) series directly connecting them ‡, and some redistribution is then made to take account of other series and circuit closing errors. The object is to secure as far as possible that the change of scale or azimuth correction imposed on unit length of any series should be roughly inversely proportional to the strength of the series §.

The scale and azimuth corrections having been decided on, revised closing errors in latitude and longitude are immediately obtainable by the use of K_1 etc., the correction to the scale and azimuth of any series being taken to be the mean of the corrections at its end. These are shown (in feet) in Plate IV.

* i.e., the values given in the computation volumes of the Dālbandin Series.

† In 1905 a correction of $-2'\ 27''\cdot 18$ was applied to all previously accepted longitudes. It has been convenient to keep all adjustment computations in terms of the old longitude and so to add $2'\ 27''\cdot 18$ to values taken from recent computation volumes. The correction has again been reapplied in Chapter I, Tables 1A and 1B, which give longitudes in modern terms.

‡ Allowance must of course be made for any circuit misclosures of scale or azimuth which may directly intervene, e.g., those at 29, 30 and 31 between 27 and 25.

§ Ordinary theory suggests that strength is inversely proportional to the square of the probable error N (see Chapter II), which gives very high weight to strong series, and very low weight to weak. This takes no account of the possibility that N itself may have been quite wrongly determined, and it has generally been thought best to weight more nearly proportionally to $1/N$ than to $1/N^2$.

An example of the form of computing the revised closing errors is shown below :—

Example of computing revised closing errors.

Series.	δS	δA	$K_1 \delta S$	+ $K_3 \delta A$ =	$\delta\phi$ (*feet*)	$K_2 \delta S$	+ $K_4 \delta A$ =	$\delta\lambda$ (*feet*)
1-26	−15	−0·8	− 4·1	− 0·6 =	− 4·7	+0·5	− 4·5 =	− 4·0
26-27	−44	−2·6	−11·4	+ 3·6 =	− 7·8	−3·0	−13·9 =	−16·9
27-29	−65	−4·6	− 3·2	−10·6 =	−13·8	+7·4	− 4·6 =	+ 2·8
1-2	00	0·0	0·0	0·0 =	0·0	0·0	0·0 =	0·0
2-28	+ 8	0·0	+ 4·0	0·0 =	+ 4·0	−0·2	0·0 =	− 0·2
28-29	+18	+0·4	+ 1·8	− 0·2 =	+ 1·6	+0·5	+ 0·8 =	+ 1·3
etc.								etc.

The above gives the change of ϕ and λ in each series. Then total changes at junction points are summed as below :—

	ϕ (*feet*)	λ (*feet*)	
Change at 1 =	0·0	0·0	
„ in 1-26 =	− 4·7	− 4·0	
	− 4·7	− 4·0	= change at 26
„ 26-27	− 7·8	−16·9	
	−12·5	−20·9	„ at 27
„ 27-29	−13·8	+ 2·8	
	−26·3	−18·1	„ at 29 (RH)
„ at 1	0·0	0·0	
„ in 1-2	0	0	„ at 2
„ 2-28	+ 4·0	− 0·2	
	+ 4·0	− 0·2	„ at 28
„ 28-29	+ 1·6	+ 1·3	
	+ 5·6	+ 1·1	„ at 29 (LH)

At 29 Sironj-terms closing error	R-L is + 40	in ϕ and + 17	in λ
From the above, change in	R-L = − 31·9	„ − 19·2	„
∴ Adjustment A closing error	= + 8·1	„ −- 2·2	„ feet

5. Final redistribution of scale and azimuth corrections.—The next process is to re-arrange the scale and azimuth corrections so as to secure zero closing errors of latitude and longitude at all circuit points by trial and error. What might have been a tedious process is considerably lightened by the practical independence of the NW. Quadrilateral, SE. Quadrilateral, South Trigon and Assam–Burma. The following is an outline of the process followed in the NW. Quadrilateral.

The circuits are first plotted as in Plate IV with the last series of one route to each circuit point misplotted in such a way as to exhibit the closing error at the scale of 100 or 250 feet to one inch. This gives a clear picture of the changes of scale and azimuth which will best close the circuits.

Attention is immediately attracted to the large (60-foot) closing errors in longitude at points 23 and 24, which are by far the largest in the whole primary triangulation. Referring to Plates II and III which show where scale and azimuth are controlled and unchangeable, it is seen that some or all of the following corrections are necessary :—

(*a*) A large decrease of scale between 1 and 8. But decreasing scale at 6 will further increase the large latitude misclosure at 18*.

* 25 feet is not outstandingly large, but it occurs in a small circuit, and so is difficult to close.

(*b*) A large increase of scale between 10 and 20, especially around 23. The weakness of 20–23 and the distance of 23 from any base-line makes this take up the greater part of the misclosure. An increase of scale at 18 also helps the latitude closure there, and increase in 20–22 helps the large longitude misclosure at 22.

(*c*) A decrease of azimuth (counter-clockwise turning) in 3–20, although this possibility is much limited by the Laplace station at 19 and the good closures at 29 and 30.

The corrections necessary to close the two 60-foot errors approximately are first allotted, the distribution between different series being such that the rate of change of correction in any series is roughly proportional to its liability to error. At this stage corrections are so far as possible applied at the numbered junction points, and the rate of change between junctions is assumed to be constant. Later (and even now in long series such as 3–19 and 19–20) corrections may have to be applied to the centres of series, additional to the mean of the corrections at their ends, but this involves a high rate of change of correction and is best avoided in the early stages. The application of a correction to any junction point (such as 5) may often adversely affect the closure of a branching series (such as 5–22). This must be taken account of, and provisionally remedied by corrections elsewhere. Small corrections can also be applied to ensure almost perfect closures of some of the circuits which already close well, such as those closing at 29, 30, 31 and 12.

After allotting corrections as above with the help of rough calculations on waste paper, the corrections are entered on a chart, and revised closing errors are formally calculated in the same way as in the example in para 4 of this Appendix. It is convenient to compute with the corrections additional to those of Adjustment A, rather than with the total corrections to Sironj terms. The revised closing errors now obtained should on the average be much smaller than the original ones of Adjustment A, but one or two errors of 10 or 20 feet are likely to be outstanding. These must now be eliminated by a repetition of the processes just gone through, care always being taken to impose on no series a total rate of change of correction badly out of proportion to its strength.

Eventually after two or three approximations, the circuit closures can be reduced to an average of about one foot, with a maximum of (say) 3 feet. The circuits are again plotted so as to exhibit these residual closing errors at the scale of (say) 10 feet to one inch, and final intermediate positions are then allotted to the circuit points and intermediate junction points. At a circuit point the final position is naturally chosen roughly between the two alternative positions, but care should be taken that the small arbitrary corrections to latitude and longitude so imposed do not differ at the two ends of any very short series, such as 18–23. (See para 6 below).

The corrections to scale and azimuth applied in the different approximations can now be totalled together and shown as final corrections to Sironj terms, as in Plate V. Corrections are assumed to change uniformly between junction points, except that when a figure is entered (in square brackets) against the centre of a series, a correction of that amount in the centre of the series (decreasing uniformly to zero at its ends) is applied in addition to the corrections indicated at the ends. Thus if corrections at the ends of a series are +10 and +20, and if [+30] is entered in its centre, the mean correction to the series is $\frac{1}{2}$ (+10+ 20+30).

With these total corrections to scale and azimuth the total corrections to Sironj-terms latitudes and longitudes are computed (as in para 4), which, with the small corrections arising from the final arbitrary distribution of the residual closing errors, give the finally adjusted values at each junction point. These are given in Tables 1A and 1B (pages 8 to 20).

6. Adjustment of intermediate stations.—Accepting the values of scale, azimuth, latitude and longitude at the junction points of series, a self-consistent set of data for each series can be obtained by adjustment between its terminal stations by the method now in use for the provisional adjustment of new series. This method is described in the departmental Handbook on Geodetic Triangulation, pages 89-93*. The basis of the method is that to satisfy four terminal conditions four unknowns must be determined, and the four selected are as follows. The series is first represented by a single line of sides called the traverse flank, which is divided into two halves†. Then the solution is made for:—

(1) η_1, the equal change in each traverse angle in the first half of the traverse.

(2) η_2, the equal change in each traverse angle in the second half of the traverse.

(3) ϵ_1, the change applied to each successive log side in the first half of the traverse‡.

(4) ϵ_2, the change applied to each successive log side in the second half of the traverse.

When these four quantities have been determined, the scale and azimuth corrections to each traverse side are known, and hence the latitude and longitude corrections to each traverse station. Suitable corrections are then given to the scale and azimuths of sides necessary to fix the remaining points of the series, and accordant corrections to their latitudes and longitudes are deduced.

An alternative way of viewing the above process is to say that the terminal scales and azimuths of the series are first adjusted to fit the imposed scales and azimuths, and that additional scale and azimuth corrections in the centre of the series are then determined in order to secure closure in latitude and longitude. If the computations by which Table 1 has been arrived at were in all respects perfect, and if the latitudes and longitudes were there given to another place of decimals, and if there had been no final arbitrary dispersal of residual closing error, these additional central corrections would be either zero or equal to the amounts entered in brackets in the centre of series in Plate V. Actually, the latitudes and longitudes in Table 1 are sometimes to a small extent (one or possibly two feet) inconsistent with the values of scale and azimuth shown in Plate V, with the result that the finally adjusted scale and azimuth in the centres of series will not be exactly identical with what has been intended. Provided the differences are not very large

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* If this complete adjustment should ever be undertaken, it will probably be profitable to review the details of this method with a view to possible numerical simplification, since as it stands it is designed for occasional use only. The formulæ given in the handbook pages 90 and 91 apply to the case where the computations available are already in final terms at one end of the series, but the necessary modifications are quite clear.

† Ordinarily the two halves should be equal, but the division should be made at an intermediate Laplace station or at any distinct change of quality (should any such exist), provided neither of the two parts so formed is unduly short.

‡ i.e., the 3rd log side is changed by $3\,\epsilon_1$. If there are 5 sides in the first half, the 8th log side is changed by $5\,\epsilon_1 + 3\,\epsilon_2$.

(as a maximum say 20 in the log or 1″ of azimuth in the centre of a primary series 100 miles or more long) that is a matter of little consequence, but larger differences would result in appreciable errors, rather than improvements, being forced into the observed angles. In a series 100 miles long a change of 20 in the centre (average 10) corresponds to a change of position of a little over 1 foot, and it is thought that in longer series discrepancies of as much as 20 will not occur, but there are some short series such as 18–23 and 50–57, where this process of adjustment could not be followed without doing serious violence to the observed angles. To avoid this, it will be necessary to adopt the following modifications of the normal procedure.

(*a*) The following pairs of short consecutive series must be adjusted as single series. The values thus obtained for the intermediate point must then be accepted for the adjustment of any other series emanating from it, instead of the values given in Table 1. Thus, adjust 1–2–3 between Table 1 values of 1 and 3. This will give scale, azimuth, latitude and longitude at 2 differing a little from those in Table 1, and which must be used instead of Table 1 values when adjusting 2–28.

10-17-18*, 30-31-25, 3-4-5, 1-2-3, 37-44-45, 39-40-41, 39-66-71, 67-68-69, 70-77-78-79, 76-82-81, 84-85-82, 81-83-84.

(*b*) The following pairs of series should be adjusted in the ordinary way, but discrepancies may arise, in which case (but not otherwise) they should be treated as in (*a*) above.

20-21-22, 48-49-50, 50-43-42†, 63-64-60, 54-58-59, 59-60-61, 74-75-76†.

If other cases unexpectedly arise, they can be treated in the same way.

(*c*) The short series 18–23 must be treated as follows. Obtain the scale and azimuth of all traverse sides in 18–23 by simple interpolation between the scale and azimuth corrections at 18 and 23. Then compute the changes of latitude and longitude which arise from them (without any solution for ϵ_1 etc.). The values so obtained at 23 should be accepted for the adjustment of 23–24 and 23–22.

Similarly 57 must be computed from 50, 80 from 79, 87 from 69, and 89 from 88.

(*d*) Pendent series such as 12-13-14, 31-32-33-34, 60-61, 64-65, and 93-96-97 must be adjusted as in (*c*), accepting Table 1 values of scale and azimuth at their far ends, but ignoring the Table 1 latitudes and longitudes there.

This process of adjustment gives changes of scale, azimuth, latitude and longitude which are to be imposed on some existing set of computations. It matters little in what terms these existing computations may be, provided they satisfy the following conditions.

(*a*) In their own terms they must be fully accurate.

(*b*) They must contain no circuit corrections of any kind. (Figural adjustments are of course necessary).

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* This will result in the inexact satisfaction of Yusuf Laplace station at 17, but it is weak and that will not matter.

† This will result in the base-line not being satisfied, but a discrepancy of 20 can be forced into the base-line extension.

(c) The differences between old and new terms must be reasonably small.

Original computation volumes generally satisfy these conditions, and so do the Sironj terms of the G.T. Volumes, but only the stations of the traverse flanks are available in Sironj terms. The final adjusted values of the G.T. Volumes, or the results of the 1916 Burma adjustment, as published in the triangulation pamphlets, contain circuit adjustments and so cannot be used.

It will often happen that the whole length of a series (or a pair of consecutive series when required) will not be obtainable already computed in any one terms. It will then be necessary to compute the smaller half in terms of the larger. This can be done by the process of computing changes, but as generally only one or two sides will be involved, it may be easier to make a full recomputation of them. Such cases occur (e.g., at the south end of 28-2) when a junction point is not on the traverse flank of the G.T. Volumes. The series 70-77-78-79 is a case which will involve more work than most.

The Dehra Dūn base-line figure has been recomputed in Sironj terms in 1936 taking account of the large deviation corrections present. These revised computations must be used when adjusting 26–27, and 27–29.

Junction points have not always been made at all Laplace stations, with the result that the adjusted azimuths eventually arrived at will not exactly satisfy the Laplace equations, although they will do so approximately. This is of no consequence provided the discrepancy is small, since Appendix VIII suggests that the probable error of a Laplace correction is $\frac{3}{4}''$. The corrections at such stations have been shown in round brackets in Plate V.

7. Karachi base-line.—The closure of the circuit errors at 18, 23 and 24 places some considerable strain on the triangulation, and when they were being adjusted a need was felt for a decrease of scale at 8 (Karāchi), which would have lessened the strain elsewhere. This suggested that the Karāchi base-line or its extension might be inaccurate, a suggestion which is slightly supported by the scale misclosures at Karāchi of triangulation emanating from other base-lines, as follows:—

Measured log length at Karāchi *minus* length based on	Sironj	via	1-5-8	=	+ ·000 0069
„ „	Chach	„	25-23-8	=	+ ·000 0166
„ „	Padag	„	12-15-8	=	+ ·000 0031
„ „	„	„	12-10-8	=	+ ·000 0087

The evidence against the base-line is not strong enough to justify its rejection, but a correction of -20 in the 7th decimal has been applied at 8, which has materially helped to close the circuits, and which is only $1\frac{1}{2}$ times the probable error of an extended base-line as estimated in Appendix VIII.

8. Dehra Dun Laplace station.—Owing to an error which was far from obvious, the azimuth correction to Sironj terms at Dehra Dūn was originally recorded as $-0''\cdot4$ instead of $-3''\cdot1$. The longitude closing errors of Adjustment A at 23, 30 and 29 were then $+73$, $+15$ and -9 feet, instead of $+70$, -2 and -2 now shown in Plate IV. Any attempt to close the large error at 23 by swinging series 3-19-20-30 over to the west was frustrated by the $+15$ closing error at 30 by which the strong series 2-29 was pulling it to the east. It was therefore suspected that the north end of 2-29 had by some error become several seconds wrong in azimuth. To test this possibility, the rather unusual azimuth station at Bowra, Kheri and Rākhi (28) was formed as described on page 56, which confirmed this suspicion as it gave a correction of $-0''\cdot 1$ * to Sironj-terms azimuths instead of about $+4''\cdot 5$ given by Adjustment A. Such an accumulation of error between Dehra Dūn and Bowra was hardly possible, and the next step was to check the Dehra Dūn Laplace station by observing longitude at the azimuth station Banog, 10 miles away. This was found to give a correction to Sironj terms of $-3''\cdot 7$ which showed the Dehra Dūn Laplace station to be wrong, enabled the Bowra station to be accepted, and helped to close the large misclosure at 23. Further examination of the Dehra Dūn Laplace station showed that the deviation correction to the actual azimuth observation had been omitted. The referring mark is at an elevation of over 5°, the deviation exceeds 30″, and the correction is $2''\cdot 7$. All the remaining angles around Dehra Dūn had been corrected †, but this one had been overlooked. The revised value of the Laplace correction is then $-3''\cdot 1$ which is meaned with $-3''\cdot 7$ at Banog to give $-3''\cdot 4$.

It is thought that this accident at Dehra Dūn provides a good example of the advantage of the informal method of adjustment now employed over a more formal solution by least squares. The latter would never have shown up the difficulty of reconciling the longitude misclosure at 23 with the wrong Laplace correction at Dehra Dūn.

9. **Nayinipiriyan Laplace station.**—The Laplace stations of the South-east Coast Series 55-64-60 show that this series has generated azimuth error as below. See Appendix II, Table 10.

Between 55 and Kallapat ...	55 miles,	$-1''\cdot 1$
Between Kallapat and Nayinipiriyān ...	55 miles,	$+1''\cdot 4$
Between Nayinipiriyān and Pātharankota	75 miles,	$-7''\cdot 0$
Between Pātharankota and Manĕgandi	30 miles,	$-2''\cdot 9$
Between Manĕgandi and Ramnad (64)	35 miles,	$+0''\cdot 4$
Between Ramnad and 60 ...	85 miles,	$-0''\cdot 8$

Internal evidence suggests that the probable accumulation of error in 100 miles of this series is about 1″ (Appendix VI), while Appendix VIII gives $0''\cdot 75$ as the probable error of a Laplace azimuth. It is clear, therefore, that either the quality of the series between Nayinipiriyān and Pātharankota is quite different from that suggested by internal evidence, or else one of these two Laplace stations is wrong. If Nayinipiriyān is rejected, the worst accumulation of error becomes $-5''\cdot 6$ in the 130 miles between Kallapat and Pātharankota, which is less unreasonable, and it was found also that the rejection of this Laplace station considerably helped the circuit misclosures at 63 and 60, which would otherwise have been very difficult to close. Principally in consequence of the latter consideration Nayinipiriyān Laplace station has been rejected.‡

* A circuit misclosure of $6''\cdot 7$ occurs at 29, so corrections of $-0''\cdot 4$ at 27 and $-0''\cdot 1$ at 28 are in strong disagreement.

† Deviation corrections to horizontal angles have been applied to base-line extensions at Dehra Dūn, Kĕng Tung, Amherst, Kalemyo, Padag, and Nāmtiāli, and to the Laplace azimuths at Dehra Dūn, Banog, Kĕng Tung, and Nāginimāra. These are places in which large corrections were to be feared.

‡ It has been included in adjustment A, but ignored in the subsequent final adjustment.

Appendix IV

ADJUSTMENT OF THE SECONDARY TRIANGULATION

1. **The South-west quadrilateral.**—The secondary triangulation forming the South-west quadrilateral is surrounded by primary series on three sides, and is divided into two by the Singi series (4–51). All these series have already been adjusted. The method followed for the adjustment of the secondary series has been exactly the same as that described in Chapter I, para 5, except that instead of first obtaining miscolsures in Sironj terms, the misclosures have been computed in terms of the 1937 values of the points 115, 105, 107, 109, 3, 2 and 114 from which the secondary series emanate, and in the process of closing the circuits by trial and error the 1937 values at these points have been held unchanged. The values finally obtained at the other junction points are given in Table 11.

The complete adjustment of the series when required, will be carried out as in Appendix III, para 6, using the values given in Table 11, and the values for 115, 105, 107, 109, 3, 2 and 114 obtained from the adjustment of the primary triangulation. The Bhīr series 46–114 will be adjusted between the values derived from its two terminal sides, and the pendent Buldāna and Kāthiāwār series will be adjusted on to the values obtained for their starting sides.

The corrections arising in the centres of series (Appendix III, para 6) will of course be larger than those arising in primary series. Large values which are anticipated are -100 in scale and $+9''\cdot0$ in azimuth in the centre of 109–110, and $-7''\cdot0$ in azimuth in the centre of 111–113.

In the series of the South-west quadrilateral as in all other secondary series, the adjusted values of azimuth should be rounded off to the nearest second, and log sides should be given to six decimals only.

2. **The North-east quadrilateral.**—The secondary triangulation of the North-east quadrilateral is surrounded by primary series on three sides and on part of the fourth (27–116). Its adjustment must for the present be postponed pending final adjustment of the Assam triangulation bounding it on the east (Point 71 in Table 1B). When this is completed the adjustment of the North-east quadrilateral can be carried out in the same way as that of the South-west quadrilateral.

3. **Other secondary series.**—The remaining secondary series are either pendent or else extend directly* between primary series which will already have been adjusted. The latter will be treated as described in Appendix III, para 6. For pendent series, the primary adjustment will give changes of scale, azimuth and position at the starting side, and the resulting changes will be calculated at their other stations.

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* The Bāgalkot series and the southern half of the Naldrug series should be treated as one, and adjusted between the Great Arc and Mangalore meridional. The north half of the Naldrug series will then be adjusted between them and the Bombay longitudinal.

TABLE 11.—*South-west quadrilateral*

	Station 111. Thīkri-Jalālabād.				
Published values	...	5·093 1439	1° 50′ 35″·1	22° 01′ 02″·77	75° 24′ 49″·98
1937 Adjustment	...	1501	36·7	02·69	49·85
	Station 112. Rewapur–Goulan Khodra.				
Published values	...	4·836 4520	45 40 27·7	22 02 55·34	76 43 52·14
1937 Adjustment	...	4697	25·5	55·30	52·11
	Station 110. Karsod–Indrāwan.				
Published values	...	5·136 3075	37 24 48·9	23 06 46·48	75 25 45·52
1937 Adjustment	...	3140	49·7	46·41	45·52
	Station 108. Mirzāpur–Wastrāl.				
Published values	...	4·749 2281	91 04 30·3	22 59 17·79	72 50 07·52
1937 Adjustment	...	2203	30·1	17·77	07·59
	Station 106. Monāba–Wāndia.				
Published values	...	4·828 9333	80 25 22·8	23 16 35·86	70 48 44·57
1937 Adjustment	...	9432	18·5	35·72	44·62

Appendix V

PROBABLE ERRORS OF LARGE ANGLES

As stated in Chapter II, para 13, a knowledge of the probable error of an observed 180° angle is necessary, in order to assess the strength of a series for carrying forward azimuth. Consideration of triangular closures and figural adjustments gives the probable error of observation of angles which average about 60°, but it is likely that 180° angles will have larger probable errors.

Of the errors by which angles are most affected, the following are likely to be equally large in 60° and larger angles :—

(1) Centering errors of the lamp or helio.

(2) Graduation errors.

(3) Casual errors of bisection.

The following sources of error, on the other hand, are likely to be more serious in a large angle.

(4) Movement of the theodolite base and stand.

(5) Lateral refraction, since it is less likely to be of the same sign in both arms of the large angle.

(6) Dislevelment due to deviation of the vertical.

In the best triangulation item (5) probably gives rise to the largest errors.

There is no satisfactory way of assessing the relative probable errors of large and small angles, but the following figures give the result of assessing it from internal evidence, namely the agreement of measures made on different zeros.

Four to six stations have been selected at random from each of 11 good primary series. At each station between 2 and 5 angles have been observed totalling between 120° and 240°. The angles have been observed on 10 zeros, generally in continuous rounds. The discrepancies between general mean and zero means give values for the probable errors of the individual angles, and also (by summing the small angles separately on each zero) for the total angle. Then at each station the probable error of the total angle may be divided by the mean of the probable errors of the small angles to give a single value of the ratio required.

In 17 cases in which the total angle is about 120°, the mean value of the ratio is $1 \cdot 12 \pm 0 \cdot 05$: for 30 cases of about 180° the mean is $1 \cdot 24 \pm 0 \cdot 05$: and for 14 cases of about 240° the mean is $1 \cdot 48 \pm 0 \cdot 08$. The general mean is 1·27 (say 1·25) implying that the probable error of a large (180°) angle averages about 1·25 times that of a small (60°) angle.

The evidence that the ratio is significantly greater than unity is conclusive, but this internal evidence cannot be expected to give a reliable value of the ratio, since of the sources of error numbered (4) (5) and (6) above, number (6) is unaffected by change of zero, and internal evidence consequently shows no signs of its effects. Similarly, movement of the stand

(No. 4) in its most serious form results in a tendency to measure all angles too small, and this also may not be reflected in the range of the zero means*. And lateral refraction is also likely to impose errors of similar sign on many zero means, although a change of sign is to be expected between day and night.

The figure 1·25 is accepted in para 15, but para 19 shows that probable errors of azimuth derived from this assumption require to be multiplied by 1½, and it is probable that some such figure as 1·8 really represents the ratio more accurately. It is noticeable that a ratio of 1·8 is approximately what would result if the small angles were considered as independently measured, ignoring the fact that they have actually been measured in continuous rounds.

* An example is given in Geodetic Report Vol. VII, page 89. Two sets of observations on 10 zeros there shown give results which are consistent but too small.

Appendix VI

PROBABLE ERRORS OF SCALE AND AZIMUTH IN DIFFERENT SERIES

1. **Summary.**—For the Indian primary and secondary series Table 12 gives the following information.

(*a*) Serial number (see below).

(*b*) Name.

(*c*) Date.

(*d*) Reference to reports in which a narrative of the work is given.

(*e*) Theodolite used (see below).

(*f*) *e*, the estimated probable error of an observed angle (see below).

(*g*) N_1, the probable error generated in the 7th decimal of the log side after 100 miles. See Chapter II, para 17.

(*h*) N_2, the probable error generated in azimuth after 100 miles. See Chapter II, para 17. The figures given include the augmenting factor of 1·5 derived in para 19.

(*i*) $N=\sqrt{N_1^2+443\ N_2^2}$.

(*j*) Classification as primary or secondary. (See below).

2. **Serial numbers.**—In Professional Paper 16, page 92, serial numbers 1 to 94 were allotted to all the Indian triangulation series, and these numbers have been used for reference in subsequent reports and also in the Triangulation Pamphlets. Further numbers 95 to 109 have been allotted to series observed subsequently. It is convenient to retain these serial numbers unchanged, except:—

(*a*) The series 31, 39, 40, 41, 42, 47, 51, 55, 59, 60, 79, 95 and 98 of previous lists are of purely local topographical value and have been excluded*. The numbers have been left blank, and series with adjacent numbers have been left unchanged.

(*b*) Series 67 (the Mong Hsat series) was re-observed in 1929–31, and the new number 104 was allotted: 104 is now retained and 67 omitted.

(*c*) Some recent series 105, 106, 108 and 109 are extensions of older series, and are more properly regarded as part of them. Details have therefore been separately included under the early number, and the later numbers have been left blank. After an interval of some years, when they have lost the special interest of recent work, it will be proper to merge them with the older work as single entries in the table.

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* In charts of the triangulation pamphlets primary and secondary series are shown by thick lines, and their stations are designated H.S. or S., in capital letters. These series may continue to be treated in the same way, for although they are of only local value the topographer can continue to regard them as errorless.

(*d*) A few old series, 7, 10, 11, 20, 23 and 52, have been divided into separate sections to indicate distinct differences of quality.

3. **Theodolites.**—The 36″, 24″, 18″ and 15″ are old micrometer theodolites. They are described in G. T. Vol. II, Appendix II. The 36″ and 24″ theodolites have generally produced work of primary accuracy, while the 18″ and 15″ have been used for secondary series.

The 12″ and 8″ micrometers are comparatively modern theodolites which have only recently become obsolete. The 12″ is a primary instrument, and the 8″ secondary. A 6″ theodolite of this type was used for series 86, but has generally been considered suitable for topographical work only.

The 5½″ and 3¾″ theodolites are modern theodolites manufactured by Messrs. Wild. The 5½″ is of primary accuracy, and the 3¾″ is suitable for secondary and topographical triangulation.

The 14″ and 12″ verniers are old instruments of secondary accuracy. Instruments not marked vernier are micrometer. It sometimes happens that a few angles of a secondary series have been observed with a better instrument near its junction with a primary series. In such cases no mention has been made of the better instrument.

4. **Probable error of an observed angle.**—The probable error of an observed (60°) angle has been deduced as follows in the different series.

(*a*) From the adjustment of figures more complicated than simple triangles (see para 13 of Chapter II) in the following series, in which such figures predominate:—5, 6, 7, 8 (see *d*), 9, 10, 11, 13, 18, 20 (west of Long. 80°), 22, 24, 25, 26, 28, 30, 32 (see *f*). 33, 34, 35, 36, 37, 38, 43, 44, 46 (see *d*), 49, 50, 52 (see *i*), 53, 54, 56, 58, 62, 63, 64, 66, 68, 69, 72, 74, 76, 77, 80, 85, 86, 101, 103, 104, 107.

(*b*) From triangular errors in the following series, in which there are very few figures other than simple triangles:—1, 3, 4, 15, 16, 19, 27, 29, 57, 61, 65*, 71, 73, 75†, 78, 81, 82, 83, 84, 87, 88, 90, 91, 92, 93, 94, 96, 97, 99, 100, 102.

(*c*) From a suitable combination of (*a*) and (*b*) above, in the following series:—2, 12, 14, 17, 21, 23 (see *d*), 70.

(*d*) In a few series the compilers of G. T. Volumes considered that the values derived as above were unduly small, taking into account the usual performance of the instrument employed, and allotted higher probable errors. In these circumstances the figures given in the G. T. Volumes have been retained.

Series 8 (in part) see G. T. Vol. VI page 88.

Series 23 (in part) see G. T. Vol. II page 352.

Series 46 (in part) see G. T. Vol. XII page 106.

(*e*) Series 20 (The North-east longitudinal) east of longitude 80°. This series was accurately observed, but the station towers were high and solid, so that it was not possible to plumb over a ground-level mark-stone. The work was prolonged over 8 separate seasons and it is estimated in G. T. Vol. VII, pages 65–67, that the probable error of log side resulting from the deflection of the towers during each interruption was 0·000 0077. An average season's

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* Quadrilaterals were observed, but the figures were not adjusted.

† *e* is taken from the triangular errors because they give the larger value.

work was 8 pairs of triangles, and these would produce an error of 0·000 0077 if $e = 1 \cdot 17$. This is far worse than the actual standard of angular measurement, but correctly reflects the inaccuracy introduced by the deflection of the towers. The value $e = 1 \cdot 20$ has now been accepted.

(*f*) Figures for series 32 (Great Indus) are given in G. T. Vol. II page 350. The figure $\epsilon_3 = 0 \cdot 242$ in group IX seems unduly small, and 0·43 has been accepted as given by the same instrument in Groups XVII and XVIII. Also 0·352 is too small a figure for Group XIX in view of the trouble recorded in G. T. Vol. III pages xii_D to xix_D, and 0·5 has been accepted for this group instead. ($e = 0 \cdot 67\ \epsilon_3$).

(*g*) The triangular errors of the Sutlej series (No. 45) give $e = 0 \cdot 23$. This is a very small value for a series with low grazing rays, and the value 0·32 has been accepted, as given by the same 36-inch instrument in other series (Group III of G. T. Vol. II, page 350).

(*h*) The East Calcutta longitudinal series (No. 48) consists of simple triangles. The triangular errors indicate that $e = 0 \cdot 26$. This is a very small figure, although not perhaps impossible for the 24-inch theodolite used. There are however two circumstances which indicate that the quality of the series is really very much worse, namely:—(i) G. T. Vol. VIII page xi_U records that the re-observation of one triangle gave differences of $-6'' \cdot 15$, $-5'' \cdot 56$ and $+4'' \cdot 68$ in the three angles: and (ii) Laplace stations show that azimuth errors of $8'' \cdot 4$, $2'' \cdot 1$ and $6'' \cdot 0$ have been generated in three sections of the series of lengths 90, 45 and 25 miles respectively. There is no way of determining a true value for e, but 1·00 has been accepted. Possibly 2·00 would be more suitable.

(*i*) In the part of the Burma Coast series (No. 52) observed in 1930–31, the figural adjustments gave the very low figure of 0·13. The sample is a small one, and it is thought that e is better represented by the figure 0·19 as given by the same instrument in series 76.

(*j*) The Buldāna series (No. 89) consists of simple triangles. The triangular errors indicate that $e = 0 \cdot 20$, which is better than most primary triangulation, while the Buldāna series was observed with an 8-inch theodolite. The figure 0·60 is typical of good work with this instrument, and has been accepted.

5. **Classification as primary and secondary.**—The figures representing the accuracy of the different series in Table 12 vary from $e = 0 \cdot 16$ and $N = 10$, to $e = 2 \cdot 27$ and $N = 447$. Series as weak as the latter are rare, but such figures as $e = 1 \cdot 00$ $N = 100$ are not uncommon, and between these and the best series there is no very clear dividing line between series which are obviously primary and those which are secondary. Wherever the line is drawn there will be border-line cases whose position is doubtful.

When considering where to draw the line between primary and secondary, attention must be paid to the type of instrument used. The figures e, N_1, N_2 and N show the accuracy which has apparently been achieved, but the methods by which these quantities have been arrived at are not infallible, especially when a series does not contain many triangles. It may be accepted as an invariable rule that a series cannot be classed as primary if it has been observed with a type of instrument which usually gives results of secondary accuracy. A few small triangular closures may bring its *e* on to the primary side of the dividing line, but it is more probable that these small closures are due to chance than that abnormal accuracy has really been obtained with a weak instrument. Similarly, a series which has been observed with a primary instrument, with the usual programme, has been classed as primary unless there is very clear evidence to the contrary, or unless some known difficulty has introduced inaccuracy.

The rules which have been followed in Table 12, are then :—

(*a*) All series observed with old 18 or 15-inch or smaller theodolites, modern 8 or 6-inch, and $3\frac{3}{4}$-inch Wild are secondary.

(*b*) All series observed with the old 36 or 24-inch, modern 12-inch, and $5\frac{1}{2}$-inch Wild are primary unless N exceeds 40. The following series have been observed with primary theodolites, but are of secondary accuracy.

(1) No. 20, the North-east longitudinal east of longitude 80°. See para 4(*e*).

(2) No. 48, the East Calcutta longitudinal. See para 4(*h*).

(3) No. 52 (*b*). Between latitudes $14\frac{1}{2}°$ and 16° four consecutive figures of the Burma Coast series have very bad side misclosures, and are undoubtedly secondary, although they were observed with a 24-inch theodolite.

(4) Nos. 27, 71, 73, 91, 93, 94 were observed with primary instruments but with a reduced number of zeros, and have not attained primary accuracy.

(5) No. 99, the Rangoon series. Observations were made from a portable trestle tower to 100-foot portable masts.

TABLE 12.—*Primary and Secondary Triangulation Series*

Serial number	Name	Date	Reference*	Instrument	e	N_1	N_2	N	Classification
1	South Pārasnāth Meridional	1836-39	S.V. XIII A	18″	2·23	147	5·52	187	Secondary
2	Budhon Meridional ...	1833-43	G.T. VII	18 & 15	1·33	70	3·34	99	Secondary
3	Amūa Meridional ...	1834-38	G.T. VII	18	1·11	69	3·12	95	Secondary
4	Rangīr Meridional ...	1834-41	G.T. VII	18 & 15	1·10	63	2·85	87	Secondary
5	Calcutta Longitudinal ...	1864-69	G.T. VI	36 & 24	0·19	8	0·39	11	Primary
6	Great Arc Meridional 24°-30°	1835-66	G.T. IV	36	0·44	22	1·02	31	Primary
7(a)	Bombay Longitudinal East of 75° ...	1862-63	G.T. XII	24	0·51	23	1·10	33	Primary
7(b)	Bombay Longitudinal West of 75° ...	1837-39	G.T. XII	15	0·79	40	1·47	51	Secondary
8	Great Arc Meridional 18°-24°	1837-41	G.T. VI	36	0·29	15	0·64	20	Primary
9	Great Arc Meridional 8°-18°	1866-74	G.T. XII	24	0·25	11	0·51	15	Primary
10(a)	Singi Meridional 21°-25°	1860-62	G.T. XIV	18	0·42	23	1·02	31	Secondary
10(b)	" " 19°-21°	1842-46	G.T. XIV	15	1·39	47	2·07	64	Secondary
11(a)	South Konkan Coast 15½°-19°	1842-44	G.T. XIII	15	2·20	83	3·42	109	Secondary
11(b)	" " Lat. 15½° ...	1866-67	G.T. XIII	24	0·27	14	0·73	21	Primary
12	Karāra Meridional ...	1843-45	G.T. VII	18 & 15	0·93	58	2·54	79	Secondary
13	North Malūncha Meridional	1844-46	G.T. VIII	18 & 15	0·88	46	2·40	68	Secondary
14	Chendwār Meridional ...	1844-46	G.T. VIII	36 & 18	0·80	45	2·16	64	Secondary
15	Gora Meridional ...	1845-47	G.T. VIII	15	0·66	41	1·90	57	Secondary
16	Calcutta Meridional ...	1845-48	G.T. VIII	18	0·79	63	2·85	87	Secondary
17	South Malūncha Meridional	1845-53	S.V. XIII A	24 & 18	1·04	65	2·80	88	Secondary
18	Khānpisura Meridional ...	1845-48	G.T. XIV	15	0·88	37	1·64	51	Secondary
19	Gurwāni Meridional ...	1846-47	G.T. VIII	24 & 18	0·79	55	2·49	76	Secondary
20(a)	North-East Longitudinal West of 80° ...	1850-51	G.T. VII	24	0·34	15	0·63	20	Primary
20(b)	North-East Longitudinal East of 80° ...	1846-51	G.T. VII	36, 24 & 15	1·20	93	4·18	128	Secondary
21	Hurilāong Meridional ...	1848-52	G.T. VIII	24 & 18	1·02	68	3·05	94	Secondary
22	North-West Himālaya ...	1848-53	G.T. III	24	0·33	17	0·63	22	Primary
23(a)	Gurhāgarh Meridional 24½°-26½° ...	1848-50	G.T. IV	18 & 15	0·53	30	1·23	40	Secondary
23(b)	Gurhāgarh 26½°-32½° ...	1859-62	G.T. IV	24	0·38	28	1·24	38	Primary
24	East Coast ...	1848-63	G.T. VI	24	0·34	20	0·84	27	Primary
25	Karāchi Longitudinal ...	1849-55	G.T. III	36	0·33	18	0·76	24	Primary
26	Abu Meridional ...	1851-52	G.T. XIV	18	0·65	33	1·58	47	Secondary
27	North Pārasnāth Meridional	1851-52	G.T. VIII	24	0·60	42	1·89	58	Secondary
28	Kāthiāwār Meridional ...	1852-56	G.T. XIV	18	0·73	42	1·76	56	Secondary
29	Gujarāt Longitudinal ...	1852-62	G.T. XIV	18	0·58	38	1·77	53	Secondary
30	Kāthiāwār Longitudinal ...	1853	S.V. XXXIV	18	1·27	64	2·43	82	Secondary
32	Great Indus ...	1853-61	G.T. III	36 & 24	0·26	14	0·66	20	Primary
33	Rahun Meridional ...	1853-63	G.T. IV	24	0·25	14	0·64	20	Primary
34(a)	Assam Longitudinal 89°-92°	1854-60	G.T. VIII	24	0·38	25	1·06	33	Primary
34(b)	" " 92°-96°	1934-36	G.R. 1934-36	5½	0·36	22	0·76	27	Primary
35	Cutch Coast ...	1855-58	G.T. XIV	18	0·74	41	1·94	58	Secondary
36	Kashmir Principal ...	1855-60	S.V. VII	14 vernier	0·89	53	1·87	66	Secondary
37	Jogi-Tila Meridional ...	1855-62	G.T. IV	36 & 24	0·34	21	1·02	30	Primary
38	Sambalpur Longitudinal ...	1856-57	S.V. XIII	14 vernier	0·74	37	1·64	51	Secondary

* S.V. = Synoptical Volume.
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TABLE 12.—*Primary and Secondary Triangulation Series.*—(contd.)

Serial number	Name	Date	Reference*	Instrument	e	N_1	N_2	N	Classification
43	Bidar Longitudinal ...	1860–72	G.T. VI	36″ & 24″	0·22	10	0·44	14	Primary
44	Eastern Frontier and Cāchār Branch ...	1860–64	G.T. VIII	24	0·35	25	0·99	33	Primary
45	Sutlej ...	1861–63	G.T. IV	36	0·32	25	1·11	34	Primary
46	Madras Meridional and Coast	1860–68	G.T. XIII	36 & 24	0·25	12	0·60	17	Primary
48	East Calcutta Longitudinal	1863–69	G.T. VIII	24	1·00	78	3·44	106	Secondary
49	Mangalore Meridional ...	1863–73	G.T. XIII	24	0·30	14	0·62	19	Primary
50	Kumaun and Garhwāl ...	1864–65	S.V. XXXV	14 & 12 vernier	1·03	60	2·07	74	Secondary
52(a)	Burma Coast 16° –23° ...	1864–74	Gen. R. 1864–70 and 1873–74	24	0·33	19	0·72	24	Primary
52(b)	" " 14½°–16° ...	1876–77	Gen. R. 1876–77	24	0·91	52	2·01	67	Secondary
52(c)	" " 10½°–14½° ...	1877–82	Gen. R. 1877–78 and 1879–82	24	0·29	16	0·56	20	Primary
52(d)	" " 10° –10½° ...	1930–31	G.R. VII	12	0·19	11	0·40	14	Primary
53	Jubbulpore Meridional ...	1864–67	G.T. VI	36	0·21	10	0·45	14	Primary
54	Madras Longitudinal ...	1865–73	G.T. XIII	24	0·24	13	0·56	18	Primary
56	Brahmaputra Meridional ...	1868–74	G.T. VIII	24	0·34	22	1·02	31	Primary
57	Coimbatore No. 1 ...	1869–71	S.V. XXIX	14	1·04	98	3·64	124	Secondary
58	Bilāspur Meridional ...	1869–73	G.T. VI	36 & 24	0·23	13	0·52	17	Primary
61	Malabar Coast ...	1872–80	S.V. XXVII and XXIX	14 & 12 vernier	1·03	57	2·48	77	Secondary
62	Jodhpur Meridional ...	1873–76	G.T. IV A	24	0·25	14	0·64	19	Primary
63(a)	South-East Coast ...	1874–80	G.T. XIII	24	0·33	24	0·93	31	Primary
63(b)	Ceylon Branch ...	1875–76	G.T. XIII	24	0·28	36	0·76	39	Primary
64	Eastern Sind Meridional ...	1876–81	G.T. IV A	24	0·28	17	0·74	23	Primary
65	Siam Branch ...	1878–81	Gen. R. 1878–81	12	2·51	144	5·88	190	Secondary
66(a)	Mandalay Meridional 18°–25°	1889–95	Gen. R. 1889–95	12	0·28	17	0·51	20	Primary
66(b)	" " 25°–27°	1936–37	G.R. 1936–37	5½	0·28	15	0·54	19	Primary
68	Manipur Longitudinal ...	1894–99	Gen. R. 1894–96 and 1898–99	12	0·30	18	0·52	21	Primary
69	Makrān Longitudinal ...	1895–97	Gen. R. 1895–98	12	0·16	8	0·27	10	Primary
70	Mandalay Longitudinal ...	1899-1900	Gen. R. 1898–1901	8	1·35	135	3·48	154	Secondary
71	Manipur Meridional ...	1899-1902 & 1915-16	Gen. R. 1899–1902 & R.V. X	12	0·55	33	1·36	44	Secondary
72(a)	Great Salween 21°–24° ...	1900–11	Gen. R. 1900-01, 1902-03 & 1907-09 R.V. I and II	12	0·28	14	0·52	18	Primary
72(b)	" " 20°–21° ...	1929–31	G.R. VI & VII	12 & 5½	0·35	15	0·76	22	Primary
73	Kidarkanta ...	1902–03	Gen. R. 1902–03 and 1904–05	12 & 7	0·89	53	2·25	71	Secondary
74	Kalāt Longitudinal ...	1904–08	Gen. R. 1904–08	12	0·22	14	0·36	16	Primary
75	"Baluchistān" (Bannu) ...	1908–09	Gen. R. 1908–09	12 & 8	0·91	61	2·22	77	Secondary
76	North Baluchistān ...	1908–10	Gen. R. 1907–09 and R.V. I	12	0·19	10	0·28	12	Primary
77	Gilgit ...	1909–11	R.V. I & II	12	0·26	36	0·45	37	Primary
78	Khāsi Hills ...	1909–13	R.V. I, II & V	8	1·37	105	4·60	143	Secondary
80	Upper Irrawaddy ...	1909–11	R.V. I & II	12	0·40	25	0·58	28	Primary
81	Jaintia Hills ...	1910–11	R.V. II	8	0·67	70	3·16	97	Secondary
82	Bhir ...	1911–12	R.V. III	8	0·53	36	1·60	49	Secondary

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TABLE 12.—*Primary and Secondary Triangulation Series*—(concld.)

Serial number	Name	Date	Reference*	Instrument	e	N_1	N_2	N	Classification
83	Rānchi ...	1911-12	R.V. III	8″	1·24	97	3·63	123	Secondary
84	Villupuram ...	1911-12	R.V. III	8	0·80	60	2·70	83	Secondary
85	Sambalpur Meridional ...	1911-14	R.V. III, V & VII	12	0·16	8	0·30	10	Primary
86	Indo-Russian ...	1912-13	R.V. V & VI.	6	2·27	418	7·52	447	Secondary
87	Khandwa ...	1912-13	R.V. V	8	0·67	45	2·04	62	Secondary
88	Ashta ...	1913-14	R.V. VII & IX	8	0·71	55	2·24	72	Secondary
89	Buldāna ...	1913-14	R.V. VII	8	0·60	44	1·96	60	Secondary
90	Naldrug ...	1913-14	R.V. VII	8	0·98	54	2·50	75	Secondary
91	Nāga Hills ...	1913-14	R.V. VII	12	0·61	40	1·52	51	Secondary
92	Middle Godāvari ...	1914-15	R.V. IX	8	0·61	35	1·71	50	Secondary
93	Kohīma ...	1913-15	R.V. VII & IX	12 & 8	0·74	46	2·08	63	Secondary
94	Cāchār ...	1914-15	R.V. IX	12	0·73	54	3·04	84	Secondary
96	Madura ...	1916-17	R.V. XI	8	0·78	53	2·37	73	Secondary
97	Bāgalkot ...	1916-17	R.V. XI	8	0·47	26	1·16	36	Secondary
99	Rangoon ...	1925-27	G.R. II & III	12	0·84	46	2·07	63	Secondary
100	Kurram ...	1927-28	G.R. IV	3¾	1·41	93	3·03	113	Secondary
101	Peshāwar ...	1927-28	G.R. IV	3¾	0·53	23	0·87	29	Secondary
102	North Wazīristan ...	1927-28	G.R. IV	3¾	1·28	108	3·21	127	Secondary
103	Chittagong ...	1928-30	G.R. V & VI	5½	0·38	29	0·86	34	Primary
104	Mong Hsat ...	1929-31	G.R. VI & VII	12 & 5½	0·47	30	0·81	35	Primary
107	Dālbandin ...	1931-32	G.R. VIII	5½	0·44	17	0·69	22	Primary

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Appendix VII

FORMULÆ FOR PROBABLE ERROR OF POSITION

This Appendix gives details of the formulæ for the probable error of position at the end of a triangulation series in certain concrete cases. Cases I, II and III are calculated on the general lines given by Dr. J. de Graaff Hunter in Professional Paper No. 16, pages 106-108.

In each case O is the fixed point relative to which errors are to be measured, and A is the further end of the series. It is required to determine the probable error of position at A or at some intermediate point B. Distances x are measured from A backwards towards O (See Plate VIII, fig. 1), and distances y at right angles to OA. The error of position in the direction OA is Δx, and normal to OA it is Δy. The probable values of Δx and Δy are E_1 and E_2 respectively. See also list of symbols on page 21.

A triangulation series OA is assumed to consist of a large number, a, of approximately equal elements each of length l miles. Similarly between O and B there are b elements, and in a section BC there are $c-b$. The triangulation determines the ratio of the lengths of adjacent elements, and also the angle between them. Let the error in the measured ratio of the $(m+1)^{th}$ element to the m^{th} be ϵ_m, and let the error in the measured angle between them be η_m.

Since the probable error of scale after l miles is $N_1\sqrt{l/100}$ in the 7th decimal of the log (see Chapter II, para 17), the probable value of ϵ_m in any series (ϵ) is $\frac{N_1}{4\cdot 34\times 10^6}\sqrt{l/100}$, and η the probable value of η_m is $\frac{N_2}{2\cdot 06\times 10^5}\sqrt{l/100}$ (in circular measure).

As determined in Chapter II, para 19, the formulæ given below require an empirical augmenting factor of $1\cdot 33$*. They then give results which on the average are consistent with the actual circuit closures of the Indian triangulation.

Case I. See Plate VIII, fig. 1. Base and Laplace station at O, assumed errorless, and none at A. $OA = 100S$ miles. Let PP′ be the p^{th} element, numbering from O. Then the total error of P′ relative to P is $l\sum_1^{p-1}\epsilon_m$ along OA and $l\sum_1^{p-1}\eta_m$ at right angles to OA

The total error of A relative to O is given by

$$\Delta x = l\sum_{p=1}^{p=a}\sum_1^{p-1}\epsilon_m$$

$$= l\left\{a\epsilon_1 + (a-1)\epsilon_2 + \cdots\epsilon_a\right\}$$

and E_1, the probable value of Δx is

$$\epsilon l\sqrt{a^2+(a-1)^2+\cdots 2^2+1^2}$$

* Except terms involving s and t, the probable errors of measured base-lines and Laplace stations. See Cases VI, VII, VIII and XI.

$=\epsilon l a \sqrt{a/3}$, with an error of 7% if $a=10$, and less if a is greater

$$= \frac{N_1}{4\cdot 34\times 10^6}\cdot\frac{la}{10}\sqrt{\frac{la}{3}} \text{ miles}$$

$$=0\cdot 070\, N_1 S\sqrt{S} \text{ feet} \qquad (1)$$

Similarly, $E_2=1\cdot 48\ \ N_2 S\sqrt{S}$ feet $\qquad$ (2)

Case II. See Plate VIII, fig. 2. The triangulation between O and A consists of several straight series, each made up of elements of length l as before. There is a base and a Laplace station at O, assumed errorless, and none at A. Let one of the series BC be of length $100L$ miles, and let it be inclined to OA at angle θ. Let PP′ be one of its elements, let G be its mid-point, and let AG $=100R$ miles.

As in case I, the error P′ relative to P is

$$l\cos\theta\sum_1^{p-1}\epsilon_m - l\sin\theta\sum_1^{p-1}\eta_m \text{ parallel to OA}$$

$$\text{and } l\sin\theta\sum_1^{p-1}\epsilon_m + l\cos\theta\sum_1^{p-1}\eta_m \text{ at right angles}$$

and Δx, the error of A relative to O is

$$l\sum_{p=1}^{p=a}\cos\theta\sum_1^{p-1}\epsilon_m - l\sum_{p=1}^{p=a}\sin\theta\sum_1^{p-1}\eta_m$$

$$=(\epsilon_1 x_1+\cdots\epsilon_b x_b+\cdots\epsilon_c x_c+\cdots)-(\eta_1 y_1+\cdots\eta_b y_b+\cdots\eta_c y_c+\cdots)$$

and similarly Δy

$$=(\epsilon_1 y_1+\cdots\epsilon_b y_b+\cdots\epsilon_c y_c+\cdots)+(\eta_1 x_1+\cdots\eta_b x_b+\cdots\eta_c x_c+\cdots)$$

If all the series between O and A are of equal strength

$$E_1=\left\{\epsilon^2(x_1^2+\cdots x_b^2+\cdots x_c^2+\cdots 1^2)+\eta^2(y_1^2+\cdots 1^2)\right\}^{\frac{1}{2}}$$

But in the more general case when ϵ and η are different for different series

$$E_1=\left\{\Sigma\epsilon^2(x_b^2+\cdots x_c^2)+\Sigma\eta^2(y_b^2+\cdots y_c^2)\right\}^{\frac{1}{2}}$$

$$=\left\{\Sigma\frac{\epsilon^2}{l\cos\theta}\left(\frac{x_b^3-x_c^3}{3}\right)+\Sigma\frac{\eta^2}{l\sin\theta}\left(\frac{y_b^3+y_c^3}{3}\right)\right\}^{\frac{1}{2}}, \text{ since each element}$$

of x is of length $l\cos\theta$, and of y each is $l\sin\theta$. The summation is of the different series OB…, BC… and CA.

$$=\frac{1}{4\cdot 34\times 10^7}\left\{\begin{array}{l}\Sigma N_1^2\sec\theta\,(x_b-x_c)\,(x_g^2+[x_b-x_c]^2/12)\\+\Sigma\, 443\, N_2^2\operatorname{cosec}\theta\,(y_b-y_c)\,(y_g^2+[y_b-y_c]^2/12)\end{array}\right\}^{\frac{1}{2}}$$

$$=\Sigma\frac{\text{BC}^{\frac{1}{2}}}{4\cdot 34\times 10^7}\left\{N_1^2(x_g^2+[x_b-x_c]^2/12)+443N_2^2(y_g^2+[y_b-y_c]^2/12)\right\}^{\frac{1}{2}}$$

The expression for E_2 is similar.

Then E, the total probable error of position $=(E_1^2+E_2^2)^{\frac{1}{2}}$

$$=\Sigma\frac{\text{BC}^{\frac{1}{2}}}{4\cdot 34\times 10^7}\left\{N_1^2(\text{AG}^2+\text{BC}^2/12)+443N_2^2(\text{AG}^2+\text{BC}^2/12)\right\}^{\frac{1}{2}}$$

$$=\Sigma\frac{L^{\frac{1}{2}}}{4\cdot 34\times 10^{6}}\left\{(N_1^2+443N_2^2)\,(R^2+L^2/12)10^4\right\}^{\frac{1}{2}}\text{ miles}$$

$$=0\cdot 122\sqrt{\Sigma LN^2\,(R^2+L^2/12)}\text{ feet} \quad \cdots\cdots \quad (3)$$

If the triangulation does not depart unduly from the straight line OA, the separate values of E_1 and E_2 are given approximately by substituting N_1 and 21 N_2 respectively for N in (3).

If there is only one straight series OA, this case is the same as Case I, and formula (3) reduces to a combination of formulæ (1) and (2). In this case $R^2=3\times L^2/12$, so that neglect of the term $L^2/12$ would introduce an error of $12\frac{1}{2}\%$ in the result. But if there are two series OB and BA of equal length, its neglect will cause an error of only 5%. Case I will always be used for a single series, so the term $L^2/12$ may generally be neglected, and the result is

$$E=0\cdot 122\sqrt{\Sigma N^2LR^2}\text{ feet} \quad \cdots\cdots \quad (3a)$$

If the series OBCA form three sides of a square (Plate VIII, fig. 3) each of length L, and with equal values of N_1 and N_2, the probable error given by (3) is $0\cdot 122\,NL\sqrt{3L}$ which is $0\cdot 58$ times the probable error of a straight series of the same total length $3L$, and $3\cdot 0$ times that of a straight series of length L directly joining O and A.

Similarly, if two series OB and BA form two sides of a square (Plate VIII, fig. 4), the probable error is $0\cdot 80$ times that of a series of the same total length, and $1\cdot 34$ times that of a straight series joining OA. In this case a close approximation to formula (3) is obtained by using formulæ (1) and (2) for a straight series whose length is $\frac{1}{2}$ (Direct distance OA + Distance OA along the series). This approximation also gives fair results in the first example where OBCA forms three sides of a square, and it is consequently applicable whenever a curved series does not deviate unduly from a straight line.

Case III. See Plate VIII, fig. 5. Bases and Laplace stations at both O and A, assumed errorless.

In case I $\Delta x=l\left\{a\epsilon_1+(a-1)\epsilon_2+\cdots+\epsilon_a\right\}$, while the error of scale at $A=\sum_1^a\epsilon_m$

The result of adjusting the series to the base at A is that each ϵ_m is replaced by $\epsilon_m-\frac{1}{a}\sum_1^a\epsilon_m$, and Δx then becomes

$$l\left\{a\epsilon_1+(a-1)\epsilon_2+\cdots\epsilon_a\right\}-\frac{l}{a}\left\{a+(a-1)+\cdots 1\right\}\left\{\epsilon_1+\epsilon_2+\cdots\epsilon_a\right\}$$

$$=l\left\{a\epsilon_1+(a-1)\epsilon_2+\cdots\epsilon_a\right\}-l\left(\frac{a-1}{2}\right)(\epsilon_1+\epsilon_2+\cdots\epsilon_a)$$

$$=l\left\{\epsilon_1\left(a-\frac{a}{2}\right)+\epsilon_2\left(a-1-\frac{a}{2}\right)+\cdots\epsilon_a\left(1-\frac{a}{2}\right)\right\},$$

since $(a-1)/2 \doteqdot a/2$

$$\text{Then } E_1 = \epsilon l \left\{ 2\,(1^2+2^2+\cdots a^2/4) \right\}^{\frac{1}{2}}$$
$$= \epsilon l a \sqrt{a/12}$$
$$= 0\cdot035\, N_1 S\sqrt{S} \text{ feet} \quad \cdots\cdots \quad (4)$$
$$\text{and } E_2 = 0\cdot74\ \ N_2 S\sqrt{S} \text{ feet} \quad \cdots\cdots \quad (5)$$

The adjustment on the base and Laplace station at A has thus halved the probable errors given by Case I for a series of the same length.

At B the middle point of OA

$$\Delta x = l\left\{\frac{a}{2}\epsilon_1 + \left(\frac{a}{2}-1\right)\epsilon_2 + \cdots \epsilon_{a/2}\right\}$$
$$- \frac{l}{a}\left\{\frac{a}{2} + \left(\frac{a}{2}-1\right) + \cdots 1\right\}\left\{\epsilon_1 + \epsilon_2 + \cdots \epsilon_a\right\}$$
$$= l\left\{\frac{a}{2}\epsilon_1 + \left(\frac{a}{2}-1\right)\epsilon_2 + \cdots \epsilon_{a/2}\right\} - \frac{la}{8}(\epsilon_1 + \epsilon_2 + \cdots \epsilon_a)$$
$$= l\left\{\epsilon_1\left(\frac{a}{2}-\frac{a}{8}\right) + \epsilon_2\left(\frac{a}{2}-1-\frac{a}{8}\right) + \cdots \epsilon_{a/2}\left(1-\frac{a}{8}\right)\right\}$$
$$+ \frac{la}{8}(\epsilon_{a/2} + \cdots \epsilon_a)$$

$$\text{and } E_1 = \epsilon l \left\{ (1^2+2^2+\cdots[3a/8]^2) + (1^2+2^2+\cdots[a/8]^2) + (a^2/64)(a/2) \right\}^{\frac{1}{2}}$$

$$\left.\begin{aligned} &= \epsilon l a \sqrt{\frac{a}{12}\cdot\frac{5}{16}} = 0\cdot020\, N_1 S\sqrt{S} \text{ feet} \\ &\text{and } E_2 = 0\cdot41\ \ N_2 S\sqrt{S} \text{ feet} \end{aligned}\right\} \text{at B} \quad \cdots\cdots \quad (5a)$$

The probable error at the mid-point is thus 0·56 times that at the end of the series.

Also, comparing (5a) with Case I for a series of length $S/2$ it is seen that the base and Laplace closure at A have reduced the probable error at B to 0·79 of what it would have been without them · · · · · · · (5b)

It follows that if a straight series OA (Plate VIII, fig. 6) of length 100S miles, is connected to a base and Laplace station by another series AC, straight or curved, of equal length and strength, the probable errors at A are given by

$$E_1 = 0\cdot020\, N_1 2S\sqrt{2S} \text{ feet}$$
$$\left.\begin{aligned} &= 0\cdot056\, N_1 S\sqrt{S} \text{ feet} \\ &\text{and } E_2 = 1\cdot16\ \ N_2 S\sqrt{S} \text{ feet} \end{aligned}\right\} \quad \cdots\cdots \quad (5c)$$

Case IV. See Plate VIII, fig. 7. The same as Case II, but with bases and Laplace stations, assumed errorless, at both O and A.

As in Case II Δx, the error of A relative to O along OA due to unadjusted errors in the series BC, is given by

$$\Delta x = \sum_b^c (\epsilon_m x_m - \eta_m y_m)$$
$$\text{and } \Delta y = \sum_b^c (\epsilon_m y_m + \eta_m x_m)$$

Adjustment gives the condition that $\sum_1^a \epsilon_m$ and $\sum_1^a \eta_m$ are known quantities, and after adjustment ϵ_m is replaced by $\epsilon_m - \frac{1}{a}\sum_1^a \epsilon_m$ (all the series are here assumed to have equal strength).

$$\Delta x \text{ (due to BC) then becomes } \sum_b^c x_m\left(\epsilon_m - \frac{1}{a}\sum_1^a \epsilon_m\right) - \sum_b^c y_m\left(\eta_m - \frac{1}{a}\sum_1^a \eta_m\right)$$

$$= \sum_b^c \epsilon_m x_m - \frac{c-b}{a} x_{G_C} \sum_1^a \epsilon_m - \sum_b^c \eta_m y_m + \frac{c-b}{a} y_{G_C} \sum_1^a \eta_m,$$

where G_C is the mid-point of BC, and $x_{G_C} = \frac{1}{c-b}\sum_b^c x_m$

$$\text{Whence } \Delta x = \sum_b^c \epsilon_m\left(x_m - \frac{c-b}{a}x_{G_C}\right) - \frac{c-b}{a}x_{G_C}\left(\sum_1^{b-1}\epsilon_m + \sum_{c+1}^a \epsilon_m\right)$$

$$- \sum_b^c \eta_m\left(y_m - \frac{c-b}{a}y_{G_C}\right) + \frac{c-b}{a}y_{G_C}\left(\sum_1^{b-1}\eta_m + \sum_{c+1}^a \eta_m\right)$$

Similarly the part of Δx due to another series ST is given by

$$\Delta x = \sum_s^t \epsilon_m\left(x_m - \frac{t-s}{a}x_{G_T}\right) - \frac{t-s}{a}x_{G_T}\left(\sum_1^{s-1}\epsilon_m + \sum_{t+1}^a \epsilon_m\right)$$

$$- \sum_s^t \eta_m\left(y_m - \frac{t-s}{a}y_{G_T}\right) + \frac{t-s}{a}y_{G_T}\left(\sum_1^{s-1}\eta_m + \sum_{t+1}^a \eta_m\right)$$

Then considering all the series between O and A, the coefficient of $\sum_b^c \epsilon_m$ in Δx is

$\left(x_m - \frac{c-b}{a}x_{G_C}\right) - \Sigma \frac{t-s}{a} x_{G_T}$, where the summation contains one term for each series except BC.

$= x_m - \Sigma \frac{t-s}{a} x_{G_T}$, where the summation includes all the series.

$= x_m - x_Q$, where Q is the centre of gravity of all the lines forming the figure O...BC...ST...A.

The coefficient of $-\sum_b^c \eta_m$ is similarly $y_m - y_Q$, and the total value of Δx due to all the series is

$$\sum_1^a \epsilon_m (x_m - x_Q) - \sum_1^a \eta_m (y_m - y_Q)$$

$$\text{Similarly } \Delta y = \sum_1^a \epsilon_m (y_m - y_Q) + \sum_1^a \eta_m (x_m - x_Q)$$

Then $E^2 = E_1^2 + E_2^2$

$$= \epsilon^2 \sum_1^a (x_m - x_Q)^2 + \epsilon^2 \sum_1^a (y_m - y_Q)^2 + \eta^2 \sum_1^a (y_m - y_Q)^2 + \eta^2 \sum_1^a (x_m - x_Q)^2$$

$= (\epsilon^2 + \eta^2)\sum_1^a R_Q^2$, where R_Q is the distance of each element from Q.

$=(\epsilon^2+\eta^2)\Sigma(t-s)(QG_T{}^2+L_T{}^2/12)$, where the summation includes one term for each series.

$$=\frac{\epsilon^2+\eta^2}{l}\Sigma L_T(QG_T{}^2+L_T{}^2/12)$$

Whence $E=0 \cdot 122\, N\sqrt{\Sigma L\,(\rho^2+L^2/12)}$ feet . . . . (6)
where $100L$ miles is the length of each series, and 100ρ miles the distance of its mid-point from the centre of gravity of the whole.

This case differs from formula (3) in that the term $L^2/12$ is not negligible.

If the triangulation does not depart unduly from the straight line OA, the separate values of E_1 and E_2 are given approximately by substituting N_1 and $21\,N_2$ respectively in formula (6), or (6a) below.

If the series form three sides of a square as in Plate VIII, fig. 3, the result of formula (6) is to that of formula (3), without the base and Laplace station at A, as $0 \cdot 55$ is to $1 \cdot 00$. If they form two sides of a square as in Plate VIII, fig. 4, this ratio is $0 \cdot 50$ to $1 \cdot 00$. It is thus approximately true to say that closing on a base and a Laplace station halves the probable error of position in curved as well as in straight series.

As in Case II, an approximate result is obtained by applying Case III to a series whose length is ½ (Direct distance OA + Distance OA along the series).

If the series BC, ST etc., are of unequal strengths, adjustment on the base at A replaces each ϵ_m in series BC by $\epsilon_m-\frac{N_c{}^2}{\sum_1^a N_m{}^2}\sum_1^a\epsilon_m$ instead of $\epsilon_m-\frac{1}{a}\sum_1^a\epsilon_m$ as given above.

Here N_C is the value of N in the series BC, and $\sum_1^a N_m{}^2$ the sum the N^2's of all the elements between O and A. It is assumed for simplicity that the ratio N_1/N_2 is the same in all the series.

Substituting this in the expression for Δx shows that wherever the fractions $(c-b)/a$ or $(t-s)/a$ occur in the formulæ already given, they must be multiplied by the factor $N_c{}^2/\sum_1^a N_m{}^2$ or $N_T{}^2/\sum_1^a N_m{}^2$, with the result that in the final expression for Δx the point Q is the centre of gravity of all the series, each being weighted according to its N^2.

Then in the expression for E, Q must be taken as above, and the terms $(\epsilon^2+\eta^2)$ and N, being different for each series, must be taken inside the Σ. Formula (6) then becomes

$E=0 \cdot 122\sqrt{\Sigma N^2L\,(\rho^2+L^2/12)}$ feet . . . . (6a)
where 100ρ miles is the distance of the mid-point of each series from the weighted centre of gravity of the whole.

Case V. See Plate VIII, fig. 8. Base-lines and Laplace stations at both O and A, assumed errorless, are joined by two series OB_1A and OB_2A.

E_1 must first be calculated for each series separately by Case III or Case IV as applicable, giving results E_{11} and E_{12}. Then for the two series combined

$$E_1 = 1/\sqrt{1/E_{11}^2 + 1/E_{12}^2} = E_{11}\,E_{12}/\sqrt{E_{11}^2 + E_{12}^2} \quad \cdots\cdots \quad (7)$$

$$\text{Similarly } E_2 = E_{21}\,E_{22}/\sqrt{E_{21}^2 + E_{22}^2} \quad \cdots\cdots \quad (8)$$

Three parallel series can of course be dealt with in the same way.

Case VI. Measured base-lines have a probable error of s in the 7th decimal of the log, and Laplace azimuths have a probable error of t''.

(*a*) The effect on Cases I or II is that an error of $\frac{528{,}000\,sS}{4\cdot 34\times 10^6} = 0\cdot 122\,sS$ feet must be combined* with formulæ for E_1, and $2\cdot 56\,tS$ must be combined with formulæ for E_2.

The formulæ of Case I then become

$$E_1 = 0\cdot 070\,S\sqrt{N_1^2 S + 3s^2} \text{ feet} \quad \cdots\cdots \quad (9)$$

$$E_2 = 1\cdot 48\ \ S\sqrt{N_2^2 S + 3t^2} \text{ feet} \quad \cdots\cdots \quad (10)$$

(*b*) In Cases III, IV and V the errors which have to be combined with the formulæ already given are clearly $0\cdot 122\times \frac{1}{2}S\sqrt{s_1^2+s_2^2}$ and $2\cdot 56\times \frac{1}{2}S\sqrt{t_1^2+t_2^2}$ where s_1 and t_1 refer to one base, and s_2 and t_2 to the other.

If $s_1 = s_2$ and $t_1 = t_2$, the formulæ of Case III become

$$E_1 = 0\cdot 035\,S\sqrt{N_1^2 S + 6s^2} \text{ feet} \quad \cdots\cdots \quad (11)$$

$$E_2 = 0\cdot 74\ \ S\sqrt{N_2^2 S + 6t^2} \text{ feet} \quad \cdots\cdots \quad (12)$$

Case VII. *See Plate VIII, fig. 9.* Two series OB and BA, of lengths $100S_1$ and $100\,S_2$ miles, lie in a straight line with bases and Laplace stations at O, B and A.

If the bases and Laplace stations are errorless, the probable error at A is simply obtained by combining (root sum of squares) two examples of Case III, but if the three bases have probable errors s_1, s_2 and s_3, and the Laplace stations t_1, t_2 and t_3, this is not correct, since the errors s_2 and t_2 at B do not have independent effects on the two series. Let the values of N be $N_{11}\ N_{21}$ in OB, and $N_{21}\ N_{22}$ in BA.

Then, as in Case VI, it is clear that

$$E_1 = 0\cdot 035\left\{N_{11}^2\ S_1^3 + N_{12}^2\ S_2^3 + 3s_1^2\ S_1^2 + 3s_2^2\ (S_1+S_2)^2 + 3s_3^2\ \ S_2^2\right\}^{\frac{1}{2}} \text{ feet} \quad (13)$$

and

$$E_2 = 0\cdot 74\ \left\{N_{21}^2\,S_1^3 + N_{22}^2\ S_2^3 + 3t_1^2\ S_1^2 + 3t_2^2\ (S_1+S_2)^2 + 3t_3^2\ \ S_2^2\right\}^{\frac{1}{2}} \text{ feet} \quad (14)$$

If $N_{11} = N_{12} = N_1$, $N_{21} = N_{22} = N_2$, $s_1 = s_2 = s_3 = s$, $t_1 = t_2 = t_3 = t$, and $S_1 = S_2$

$$E_1 = 0\cdot 035\,S\sqrt{2\,N_1^2\,S + 18\,s^2} \text{ feet} \quad \cdots\cdots \quad (13a)$$

$$E_2 = 0\cdot 74\ \ S\sqrt{2\,N_2^2\,S + 18\,t^2} \text{ feet} \quad \cdots\cdots \quad (14a)$$

When S is small these formulæ give results $1\cdot 22$ times those obtained by combining two examples of Case VI (*b*). In India bases are never close together, but Laplace stations sometimes are, and this case cannot be ignored.

---

* i.e., root sum of squares.

If the series OB and BA lie at right angles to each other instead of in prolongation, they can be treated by combination of Case VI (b): for the effect of the scale error at B on OB will be right angles to its effect on BA.

In general, if the E_1 of OB is E_{11} (obtained by Cases III, IV or V) and that of BA is E_{12}, E_1 and E_2 for the whole series OA are given by

$$E_1 = \sqrt{E_{11}^2 + E_{12}^2 + (0 \cdot 061)^2 \left\{ s_1^2 S_1^2 + s_2^2 (S_1 + S_2)^2 + s_3^2 S_2^2 \right\}} \quad \cdot \quad \cdot \quad (13b)$$

$$E_2 = \sqrt{E_{21}^2 + E_{22}^2 + (1 \cdot 28 \quad)^2 \left\{ t_1^2 S_1^2 + t_2^2 (S_1 + S_2)^2 + t_3^2 S_2^2 \right\}} \quad \cdot \quad \cdot \quad (14b)$$

and in the case of three series in sequence

$$E_1 = \sqrt{E_{11}^2 + E_{12}^2 + E_{13}^2 + (0 \cdot 061)^2 \left\{ s_1^2 S_1^2 + s_2^2 (S_1 + S_2)^2 + s_3^2 (S_2 + S_3)^2 + s_4^2 S_3^2 \right\}} \text{ etc.,} \quad \cdot \quad (13c)$$

Case VIII. See Plate VIII, fig. 10. As in Case VII two series OB and OA lie in a straight line, with bases and Laplace stations at O and A. A third base and Laplace station at C are connected by a branch series BC.

Let the lengths of OB, BA and BC be S_1, S_2 and S_3 respectively. Let N_1 and N_2 of all three series be equal: or if BC is of different strength replace S_3 by $S_3\,(N_{BC}^2/N_{OA}^2)$ in the formulæ which follow.

The following argument is loose, but gives results which will be sufficiently accurate in practice, as is seen from the two extreme examples given below.

Ignoring the probable errors of the observed base-lines, the probable error of scale at B is

$$N_1 \sqrt{\frac{1}{1/S_1 + 1/S_2 + 1/S_3}} = N_1 \sqrt{\frac{S_1 S_2 S_3}{S_1 S_2 + S_2 S_3 + S_3 S_1}}$$

This is zero if $S_3 = \text{zero}$, and allowance for the fallibility of base measurement is made by replacing it by

$$\sqrt{\frac{N_1^2 S_1 S_2 S_3}{S_1 S_2 + S_2 S_3 + S_3 S_1} + s^2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (15)$$

Similarly the p.e., of azimuth

$$= \sqrt{\frac{N_2^2 S_1 S_2 S_3}{S_1 S_2 + S_2 S_3 + S_3 S_1} + t^2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (16)$$

Case VII may now be applied using the expressions (15) and (16) for the probable errors s_2 and t_2 of the base and Laplace station at B.

Substituting in (13) the probable value of Δx at A is given by

$$E_1 = (0 \cdot 035) \left[N_1^2 S_1^3 + N_1^2 S_2^3 + 3s^2 S_1^2 + 3s^2 S_2^2 + 3 (S_1 + S_2)^2 \left\{ \frac{N_1^2 S_1 S_2 S_3}{S_1 S_2 + S_2 S_3 + S_3 S_1} + s^2 \right\} \right]^{\frac{1}{2}} \quad \cdot \quad \cdot \quad (16a)$$

Now take two concrete cases :—

(1) $S_1 = S_2 = S$. $S_3 = 0$. This is simply Case VII.

The expression (16a) becomes $0\cdot035S\sqrt{2N_1^2S+18s^2}$ which agrees with (13a).

(2) $S_1=S_2=S$. $S_3=\infty$. This is simply Case VI (b) (formula 11) with length of series $2S$, for which the correct result is $0\cdot035S\sqrt{8N_1^2S+24s^2}$

The expression (16a) becomes $0\cdot035S\sqrt{8N_1^2S+18s^2}$. This is nearly correct, since the error is only 12% when N_1^2S is small compared with $3s^2$, and it will be less in any ordinary case. Further, this formula will generally only be used when S_3 is short.

The rule may therefore be followed that Case VII may be employed with s_2 and t_2 given by formulæ (15) and (16).

Case IX. See Plate VIII, fig. 11. As in Cases I and II there is a base and a Laplace station at O only, but the triangulation consists of two series OA and AA′ of unequal strength. The probable error of position at A is given by Cases I or II and VI (a). The probable error of scale and azimuth at A is obtained from the formulæ given in Chapter II, para 17, and the error of position generated in AA′ is then given by Case VI (a), using these scale and azimuth probable errors for s and t. The total probable error of position is then the root sum of the square of the errors generated in each section*.

Case X. See Plate VIII, fig. 12. Two secondary series OA and O′A each of length $100S$ miles lying parallel and close to each other, emanate from a primary series (assumed errorless) at O and O′, and are connected together at A. Let OA and O′A be of equal strength.

Considering the two series as separate examples of Case I, the probable error of the mean position of A is given by

$$E_1 = \frac{1}{\sqrt{2}}0\cdot070\,N_1S\sqrt{S}\text{ feet}$$

$$E_2 = \frac{1}{\sqrt{2}}1\cdot48\quad N_2S\sqrt{S}\text{ feet}$$

Allowance must however be made for the increased strength derived from the connection between the two series in scale and azimuth. The last two paras of Case III show that this can be done by multiplying by 0·79. The final result is then

$$E_1=0\cdot039\,N_1S\sqrt{S}\text{ feet} \qquad (17)$$

$$E_2=0\cdot83\quad N_2S\sqrt{S}\text{ feet} \qquad (18)$$

If the scale and azimuth of the primary series near O and O′ have probable errors of s and t, formulæ (17) and (18) must be combined (root sum of squares) with quantities $0\cdot122\,sS$ and $2\cdot56\,tS$ feet respectively.

The probable error of the relative positions of B and B′, the mid-points of OA and O′A may be obtained as follows :—

Regarding OB and O′B′ as two separate examples of Case I, the relative probable errors are

Parallel to OA $\quad 0\cdot070\sqrt{2}\times N_1\frac{S}{2}\sqrt{\frac{S}{2}}$ feet

* This last line is not strictly correct since the errors generated in the two sections are not entirely independent. It is, however, a good approximation when the series AA′ is considerably the weaker of the two.

Normal to OA $\quad 1\cdot 48 \sqrt{2} \times N_2\frac{S}{2}\sqrt{\frac{S}{2}}$ feet

The triangulation BAB′ independently determines the relative errors with equal precision, for the absence of base and Laplace station at A affects BAB′ as a whole and does not much affect the relative error of B and B′, which are close together. No factor is required on account of the scale and azimuth connection at A, as the unadjusted scales and azimuths at B and B′ are not very much affected thereby.

Final results for the relative probable error of B and B′ are then

$$\left.\begin{array}{ll} \text{Parallel to OA} & 0\cdot 025\, N_1 S\sqrt{S} \text{ feet} \\ \text{Normal to OA} & 0\cdot 52\ \ N_2 S\sqrt{S} \text{ feet} \\ \text{Total} & 0\cdot 025\, N\, S\sqrt{S} \text{ feet} \end{array}\right\} \quad \ldots \quad (18a)$$

Case XI. See Plate VIII, fig. 13. A secondary series of length $100S$ miles joins two primary series which are assumed errorless in scale, azimuth and position. The probable error at B, the mid-point of the secondary series is required.

Regarding OB and O′B as separate examples of Case I, the probable error of the mean position of B is given by

$$E_1 = \frac{1}{\sqrt{2}}\, 0\cdot 070\ N_1\frac{S}{2}\sqrt{\frac{S}{2}} = 0\cdot 018\, N_1 S\sqrt{S} \text{ feet}$$

$$E_2 = 0\cdot 37\ \ N_2 S\sqrt{S} \text{ feet}$$

Some allowance must be made for the increased strength derived from the connection in scale and azimuth at B as in formula ($5b$), and a factor of $0\cdot 79$ may be introduced on this account. Final results are then

$$E_1 = 0\cdot 014\, N_1 S\sqrt{S} \text{ feet} \quad \ldots \quad (19)$$

$$E_2 = 0\cdot 29\ \ N_2 S\sqrt{S} \text{ feet} \quad \ldots \quad (20)$$

If the primary series are regarded as errorless in position but fallible in scale and azimuth, let the probable errors be $s_1\ t_1$ at one end and $s_2\ t_2$ at the other.

Then formulæ (9) and (10) of Case VI give probable errors at B as determined from each end separately, but allowance for scale and azimuth closure must be made by multiplying each term containing N^2 by $(0\cdot 79)^2$.

$$\left.\begin{array}{rl} \text{Then } E_{11} & = 0\cdot 070\, \frac{S}{2}\sqrt{0\cdot 62\, N_1^2\, \frac{S}{2} + 3s_1^2} \\ & = 0\cdot 035\, S\sqrt{0\cdot 31\, N_1^2 S + 3s_1^2} \text{ feet} \\ E_{12} & = 0\cdot 070\, \frac{S}{2}\sqrt{0\cdot 62\, N_1^2\, \frac{S}{2} + 3s_2^2} \\ & = 0\cdot 035\, S\sqrt{0\cdot 31\, N_1^2 S + 3s_2^2} \text{ feet} \end{array}\right\} \quad \ldots \quad (20a)$$

and E_1, the combined error, $= E_{11} E_{12}\sqrt{E_{11}^2 + E_{12}^2}$ $\quad \ldots \quad (20b)$

Similarly $E_{21} = 0\cdot 74 S\sqrt{0\cdot 31\, N_2^2 S + 3t_1^2}$ feet, etc., $\quad \ldots \quad (20c)$

If $s_1 = s_2 = s$ and $t_1 = t_2 = t$

$$E_1 = \frac{1}{\sqrt{2}}\, 0\cdot 035 S\sqrt{0\cdot 31 N_1^2 S + 3s^2} \text{ feet} \quad \ldots \quad (20d)$$

$$\text{and } E = \frac{1}{\sqrt{2}}\, 0\cdot 035 S\sqrt{0\cdot 31 N^2 S + 3\,(s^2 + 443t^2)} \text{ feet} \quad \ldots \quad (20e)$$

Appendix VIII

PROBABLE ERRORS OF BASE-LINES AND LAPLACE AZIMUTHS

A. Base-lines

The error of a base-line may be considered under three headings:—

(*a*) The error of the standard of length.

(*b*) The error of measurement.

(*c*) The error of extension.

The first of these is considered in Appendix X. It will have an equal effect on all the old base-lines, and is not considered further in this Appendix.

The probable error of measurement with the compensated bars is considered in G. T. Vol. I, pages 92 to 97, where it is concluded to be $\pm 2 \cdot 6$ in 10^6. The probable error of measurement of new invar wire base-lines is very small (apart from errors of standard common to all the wires used), and in Geodetic Report 1934, page 13, the agreement of measures by different wires is shown to give ± 1 in $3 \cdot 7 \times 10^6$, 1 in $5 \cdot 2 \times 10^6$ and 1 in $2 \cdot 3 \times 10^6$, for the probable errors of measurement of three base-lines. Geodetic Report 1933, pages 40 and 41, investigates the case of three base-lines measured in 1932–33 when larger changes occurred in the lengths of the wires, and concludes that error of measurement will not have exceeded 1 in 300,000. The probable error may be quoted as ± 1 in 10^6.

The error generated in the extension from the short side (6 to 10 miles) actually measured to the normal 20 or 30-mile side of the triangulation is generally likely to be larger than that of measurement, especially in modern base-lines. Geodetic Report 1934, page 14, investigates three base-lines and finds $\pm$ 1 in 650,000, 1 in 550,000 and 1 in 580,000, giving a mean of $\pm$ $1 \cdot 7$ in 10^6 for the probable error of the extension. G. T. Vol. II, page 243, gives $\pm$ $3 \cdot 6$ in 10^6 for the probable error of extension of the Sironj Base, and G. T. Vol. XII, page 145_A, gives $\pm$ $1 \cdot 1$ in 10^6 for the extension of the Cape Comorin Base.

The method of Geodetic Report 1934 has been applied to two other old bases (selected on account of the simplicity rather than the strength of their extensions), and gives $\pm 1 \cdot 2$ in 10^6 for Karāchi and $\pm 1 \cdot 6$ in 10^6 for Calcutta.

A probable error of $\pm 2 \cdot 0$ in 10^6 may be accepted for the extension of all base-lines both old and new. For new base-lines very little need be added for the probable error of measurement, but for old base-lines $\pm 2 \cdot 6$ in 10^6 must be added on this account, bringing the total up to $\pm 3 \cdot 3$ in 10^6. Expressed in units in the 7th decimal of the log, these figures are ± 9 for new base-lines and ± 14 for old, and it will suffice to accept ± 12 for all.

It is thus concluded that the probable error of an extended base-line, apart from error in the standard of length, is $\pm \cdot 0000012$ in the log.

B. Laplace Azimuths

The error of the azimuth deduced at a Laplace station must be considered under two headings :—

(*a*) The error of the azimuth observation.

(*b*) The error arising from the measured (or assumed) value of the deviation of the vertical.

As a guide to the probable error of the azimuth observation there is the obvious fact that it cannot be less than the probable error of an ordinary horizontal angle measured with the same instrument, although it may possibly be greater. Table 12 (Appendix VI) shows that $\pm 0''\cdot 3$ is typical for the probable error of an observed angle in good primary series, such as contain most of the Laplace stations.

At a few stations astronomical azimuths have been observed on two independent occasions, and at a few others it has been observed independently to two different referring marks. Results are :—

(*a*) Kaliānpur	1836	190° 27′ 06″·20		
	1898–99	06·37		
(*b*) Kēng Tung	1931	130 27 22·1	} One R.M. to east and one to west.*	
		21·2		
(*c*) Nāginimāra	1933–34	87 57 16·7		
	1934–35	14·3	} One R.M. east and one west.	
		17·8		
(*d*) Tatalia	1934–35	206 43 13·7	} One R.M. east and one west.	
		09·5		

In the following examples, astronomical azimuth has been observed at two stations very close together, so that the deduced azimuth corrections should be identical.

(*e*) At Dehra Dūn the deduced correction to Sironj terms $=3''\cdot 1$ (see Appendix II, Table 10), and at Banog it is $-3''\cdot 7$.

(*f*) At Karāchi observatory and at Karāchi Base south end the corrections are $-2''\cdot 4$ and $-2''\cdot 2$ respectively.

(*g*) At Kudankulam and Rādhāpuram the corrections are $-7''\cdot 8$ and $-7''\cdot 3$.

The above seven examples suggest $\pm 1''\cdot 0$ as the probable value of the difference of two independent astronomical azimuths, or $\pm 0''\cdot 7$ as the probable error of a single one. These figures are considerably influenced by the large discrepancies at Tatalia and Nāginimāra, which are believed to be non-typical†, and a smaller figure appears to be more probable.

The probable error arising from the measured (or assumed) deviation of the vertical is likely to be very small when the longitude station is identical with the azimuth station. A figure such as $\pm 0^s\cdot 03 \cot \phi$, or $\pm 0''\cdot 2$ (of arc) in northern India and less further south, is probably correct. At

* Geodetic Report Vol. VII, page 91, gives slightly different figures which do not include the correction on account of the elevation of the referring mark. See Chapter III, para 22.

† Modern azimuth observations have been more rapidly made than the older ones, and so may have been more affected by lateral refraction. Kēng Tung, Nāginimāra and Tatalia are the only modern azimuths involved in Laplace stations. Tatalia and Nāginimāra are also especially liable to error on account of the constancy of sign of the bubble correction.

non-identical stations there is more doubt. Its amount is shown by the variations in the corrections to the triangulation deduced at the following groups, but it must be remembered that this variation includes also error in the azimuth observation and possibly some change of error in the triangulation itself :—

(*h*) Bolārum, Dāmargīda and Kodangal give $-1''\cdot6$, $-1''\cdot3$ & $-3''\cdot7$.

(*i*) Vizagapatam Base north end and Sānjib give $-5''\cdot3$ & $-5''\cdot2$.

(*j*) Māndvi, Dhauleshvaram and Alsunda give $-4''\cdot3$, $-3''\cdot2$ & $-4''\cdot2$.

(*k*) Karāchi and Kārothol give $-2''\cdot4$ & $-2''\cdot6$.

(*l*) Anandalamalai and St. Thomas's Mount give $-8''\cdot6$ & $-6''\cdot3$.

(*m*) Manĕgandi and Ramnad give $-17''\cdot0$ & $-16''\cdot6$.

(*n*) Nagarkhāna and Semu Tan give $+1''\cdot8$ & $+3''\cdot3$.

These seven examples suggest $\pm0''\cdot9$ as the probable value of the difference between two independent Laplace corrections including all sources of error. That is $\pm0''\cdot45$ for the mean of a group of two, or $\pm0''\cdot6$ for a single one. These examples also are to some extent non-typical, since stations such as those in examples (*h*), (*i*) and (*j*) would have been considered too doubtful for acceptance by themselves. It is also to be noted that the stations likely to be burdened by the largest error arising from deviation of the vertical have generally been measured in groups of two or three, and so are less liable to error from the observed astronomical azimuth.

Taking these considerations into account a probable error of $0''\cdot5$ for a Laplace azimuth is accepted at the beginning of Chapter II para 19, but the comparison there made between the actual and probable Laplace closures shows that the calculated probable errors require an augmenting factor of 1·5. This factor appears to be necessary in the probable error of the Laplace station as well as in that of the triangulation (see para 19), so the value finally accepted for the probable error of a Laplace azimuth is $\pm0''\cdot75$.

Appendix IX

APPLICATION OF APPENDIX VII TO INDIA

1. Symbols

E_1 = probable error in feet in the length of a series.

E_2 = probable error in feet normal to the length of a series.

$E = \sqrt{E_1^2 + E_2^2}$ = total probable error.

E_N = north-south component of E_1 and $E_2 = (E_1^2 \cos^2 A + E_2^2 \sin^2 A)^{\frac{1}{2}}$

E_E = east-west component of E_1 and $E_2 = (E_1^2 \sin^2 A + E_2^2 \cos^2 A)^{\frac{1}{2}}$

A = azimuth of the series.

The following paragraphs give details of the application of the different "Cases" of Chapter II, para 18, to the Indian triangulation, to give probable errors of position relative to the origin at Kaliānpur. The probable errors of base-lines and Laplace stations are taken to be $s = 12$ and $t = 0''\cdot 75$ respectively, as derived in Appendix VIII.

The numbered junction points by which series are described below are shown on Plate I, and base-lines and Laplace stations are shown on the frontispiece.

2. N.W. Quadrilateral

(*a*) *Dehra Dūn 27.*—This point is reached from Kaliānpur by two routes of similar strength, 1-26-27 and 1-2-29-27. Use Case V and VIb.

Apply Case III to 1-26-27. $\left.\begin{array}{l} E_1 = E_N = 9\cdot 2 \\ E_2 = E_E = 8\cdot 9 \end{array}\right\}$ . . . . . (*1*)

Apply Case IV to 1-2-29-27, ignoring the weak Laplace station at 28.

$\left.\begin{array}{l} E_1 = E_N = 11\cdot 4 \\ E_2 = E_E = 10\cdot 0 \end{array}\right\}$ . . . . . (*2*)

Combining the two routes and allowing for s and t gives

$E_N = 9\cdot 3$, $E_E = 9\cdot 7$ at 27 . . (*3*)

Since 2-29 is a stronger series than 1-27, 29 will be very slightly more strongly fixed than 27, and at 29 we may accept

$E_N = E_E = 9\cdot 0$ . . . . . . (*4*)

(*b*) *Karāchi 8*, (*Preliminary*).

For E_1 apply Case III and VIb. Then $E_1 = 16\cdot 6$.

For E_2 apply Case VII, allowing for Laplace stations, at 5 and 7.

$E_2 = 11\cdot 5$

Then $\left.\begin{array}{l} E_N = E_2 = 11\cdot 5 \\ E_E = E_1 = 16\cdot 6 \end{array}\right\}$ . . . . . (*5*)

These figures are preliminary only, since some strength is derived by the route 1-29-25-8.

(*c*) *Deesa 5.* By Case III, Note (*2*). E_1 at middle of 1-8 $= 0{\cdot}56 \times 16{\cdot}6 = 9{\cdot}3$.

For E_2 apply Case III and VIb. $E_2 = 6 \cdot 0$.

$$\text{Then } \left.\begin{array}{l} E_N = E_2 = 6\cdot0 \\ E_E = E_1 = 9\cdot3 \end{array}\right\} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \qquad (6)$$

(*d*) *Chach 25.* Consider first its fixing from 29.

E_1 by Case III and VIb, and E_2 by Case I and VIa. (The Dehra Dūn base and Laplace station are at 27, not 29, but this adds little to the probable errors of scale and azimuth at 29).

$E_1 = 5 \cdot 4$ and $E_2 = 11 \cdot 7$, whence $E_N = 9 \cdot 6$ and $E_E = 8 \cdot 6$ in 29-25. Combining this with the p. e. at 29, as given by (*4*), gives:—

$E_N = 13 \cdot 2$, $E_E = 12 \cdot 4$ at 25 via 1-29-25 · · · · · · (*7*)

Now consider its fixing from 8. The Padag base at 12 is connected to the series 8-25 at 10 and 24. By Case VIII it is equivalent to observed bases at 10 and 24 with probable errors of $17 \cdot 3$ and $17 \cdot 8$ respectively. Then for E_1 apply Case VII, and $E_1 = 12 \cdot 0$ in 8-25.

For E_2, apply Case III and VIb to 8-18, and $E_2 = 5 \cdot 6$.

Then apply Case I and VIa to 18-25, and $E_2 = 13 \cdot 2$.

Combining these gives $E_2 = 14 \cdot 3$ in 8-25.

Whence $E_N = 12 \cdot 6$ and $E_E = 13 \cdot 8$ in 8-25 · · · · · · · (*8*)

Combining (*8*) and (*5*) gives for the p. e. at 25 via 1-8-25.

$E_N = 17 \cdot 1$, $E_E = 22 \cdot 6$ · · · · · · · · · (*9*)

Then final results at 25, are obtained by combining (*7*) and (*9*) which give:—

$E_N = 10 \cdot 5$, $E_E = 10 \cdot 8$ at 25 · · · · · · · (*10*)

(*e*) *Karāchi 8* (*Final*)

Combining (*8*) and (*7*) gives for the p. e. of 8 via 1-29-25-8.

$E_N = 18 \cdot 2$, $E_E = 18 \cdot 5$.

Combining this with (*5*) as given by 1-8, gives for the final p. e.

$E_N = 9 \cdot 8$, $E_E = 12 \cdot 4$ at 8 · · · · · · · (*11*)

(*f*) *Padag 12.* There is a base-line at 12, but no Laplace station. Treat the Laplace station at 13 as if it was at 12. There are two routes from 8 to 12, both bent. Treat them as one straight route with a mean value of N_1 and N_2.

Then by Case III and VIb $E_1 = 5 \cdot 2$ and $E_2 = 6 \cdot 0$ in 8-12.

Whence $E_N = 5 \cdot 4$ and $E_E = 5 \cdot 8$.

Combining this with (*11*) gives

$E_N = 11 \cdot 2$, $E_E = 13 \cdot 7$ at 12 · · · · · · · (*12*)

(*g*) *Kūh-i-Malik Siāh 14.* Treating the Laplace station at 13 as if it was at 12, as above, Case I and VIa gives.—

$E_N = E_2 = 7 \cdot 9$ and $E_E = E_1 = 8 \cdot 1$ in 12-14.

Combining this with (*12*) gives

$E_N = 13\cdot6$, $E_E = 16\cdot2$ at 14 . . . . (*13*)

(*h*) *Indo-Russian junction 34.* From 25 to 33 the series is strong but thenceforward it is very weak. Case IX.

In 25-33 Case I and VIa gives $E_1 = 10\cdot8$, $E_2 = 5\cdot7$.

Whence $E_N = 9\cdot8$ $E_E = 7\cdot3$ . . . . (*14*)

At 33 probable error of scale $= 52$, and of azimuth $1''\cdot1$.

Then in 33-34, Case I and VIa gives $E_1 = 65$, $E_2 = 25$.

Whence $E_N = 62$, $E_E = 33$ . . . . (*15*)

Combining (*10*) and (*14*) gives $E_N = 14\cdot4$, $E_E = 13\cdot0$ at 33 . . (*16*)
and combining (*16*) and (*15*) gives $E_N = 63$, $E_E = 35$ at 34 . . (*17*)

3. Other Primary Series

The previous paragraph sufficiently illustrates the method used, and it is not necessary to go into much further detail.

Bīdar (47) is calculated by Case V and VIb, the two routes being 1-46-47 (Case III) and 1-35-49-47 (Case IV). Bangalore (54) is calculated from Bider by Case VII, Note 2, the accumulation of error in 47-54 being obtained by Case V with 3 routes:—47-54 direct (Case III), 47-49-55-54 (Case IV), and 47-52-53-54 (Case III for 47-52 and Case IV for 52-53-54). Cape Comorin (61) is similarly calculated by combination of Case VII, Note 2 (3 series), and Case V (54-59-61 and 54-55-64-61). Vizagapatam (43) is calculated from Bider by Case III and VIb. Calcutta (39) is calculated by the combination of two routes:—1-36-39 (Case III and VIb for E_1, and Case VII for E_2), and 43-41-39 (Case III and VIb).

The connection between Calcutta and Burma is at present very weak (See Appendix VI para 4 (*h*) and Chapter IV, para 26). Its strengthening is in the programme for 1939–41, and the probable errors now calculated are based on the assumption that 66-69 has become a strong series, with a base-line at 69.

The probable error at 69 is then only slightly greater than that at 39. Nāmtiāli (75) is calculated from 69 by Case III and VIb, the circuit 70-73-74-79-70 being considered to be replaced by a straight diagonal series of equal strength. Moulmein (98) is calculated by the combination of two routes:—69-91-98 (Case III and VIb, and Case VII) and 75-76-90-98 (Case VII, and Case VIII to include the Kēng Tung Base). Mergui (101) is calculated from Moulmein by Case III and VIb, and the Siamese junction at 103 is calculated from Mergui by combination of Case I and Case VII, Note 2. Ḵalemyo 88 is calculated from 69 by Cases III and VIb (E_1) and I and VIa (E_2).

4. Secondary Series

The secondary triangulation of the North-east quadrilateral is surrounded by primary triangulation on its south and west sides. On the north there is primary triangulation from 27 to 116, and at 71. The probable error at 117 is obtained by combination of Case I and VIa from 116 with

Case X from 36 (extended to include three series by multiplication by $\sqrt{2/3}$), and that at 119 in a similar way. Case X (two series only) similarly gives the probable error at 118. Case XI gives the probable errors in the centres of the Calcutta meridional, Rangīr, and Budhon (north and south halves) series, which run direct between primary series. The relative errors between the centres of adjacent parallel series are given by Case X, Note (2).

In the South-west Quadrilateral, for the purpose of exhibiting probable errors, the short length of the Bombay longitudinal series (47-51) between Poona and longitude 75° is regarded as primary. The South-west Quadrilateral then consists of a number of secondary series, surrounded by primary triangulation on three sides. The probable error at the centre of the Singi series is given by Case XI, and at the centre of the Khānpisura series it is given by Case XI reinforced by the Khandwa series (Case I and VIa). At the south end of the Abu Meridional series it is given by Case I and VIa, with slight reinforcement from the adjacent Singi series. Case I and VIa gives the probable error at different points on the Kāthiāwār meridional and longitudinal series and at the south end of the Buldāna series, and Case XI is used for the centres of the Bhīr and Cutch Coast series.

Other isolated secondary series which occur in several parts of India can be treated by Case I and VIa when they are pendent series, or by Case XI when they close on primary triangulation at either end.

APPENDIX X

THE STANDARDS OF LENGTH

1. **Summary.**—The lengths of the base-lines measured between 1831 and 1882, and consequently the lengths of the sides of the whole triangulation, are expressed in Indian feet or tenths of the standard known as Bar A at 62° F, which was used for the standardization of the compensated bars employed for base measurement. Bar A has not been used in connection with modern base-lines (1930–35), for which the invar wires have been compared with modern metre standards.

The old triangulation forms a self-consistent whole irrespective of any knowledge of the length of Bar A, although a knowledge of the length of the standard is necessary if the triangulation is to be used for any study of the earth's figure. Now that modern base-lines are being included, a knowledge of the length of the old standard is further necessary in order that modern base-lines may be expressed in Indian feet.

2. **Clarke's comparisons.**—In 1865 Captain Clarke* made comparisons between the standards of length of various countries, and included in his comparisons the Indian bars I_s and I_B. These were compared with bar A in India in 1867,† with the result that the lengths of all three bars can be expressed in terms of the British standard yard of that date. The results are given in Table 13, column 2. Captain Clarke also included two standard metres in his observations, but it is not possible to relate them to the modern International metre‡, and his observations do not suffice to give the lengths of the bars in modern metres.

3. **Yard-metre ratio.**—Trustworthy comparisons between the yard and the metre have been made on two occasions, namely by Benoit in 1894 (1 metre = 39·370113 inches) and by Sears in 1926 (1 metre = 39·370147 inches). The difference is small, but the 1926 value is believed to be the better. These results enable the Indian standards to be expressed in terms of the International metre, provided the British standard yard has been stable between 1865 and 1894 or 1926.

4. **Stability of the yard.**—The stability of the yard has been questioned. It has been suggested that it and the copies with which it is periodically compared may have been shortening, especially in the early years of their existence.§ Force is lent to this suggestion by the behaviour of a copy (P.C. VI) which was made later than the standard yard (in 1879), and which shortened considerably during the first 30 years of its existence. When trying

* "Comparisons of the standards of length", by Captain A. R. Clarke R.E., F.R.S., 1866.

† G.T. Vol. I, pages 1-32.

‡ The difficulties are outlined in two letters from the Director of the National Physical Laboratory. (Nos. 789/SWA of 16-7-37 and 1-9-37. Geodetic Branch file 47-R of 1937). Thanks are due to Mr. S. W. Attwell M.B.E. of the N.P.L. for the interest which he has taken in the Indian standards, and for the help which he has given in elucidating details of the old comparisons.

§ Reports by the Board of Trade on the "Comparisons of the Parliamentary Copies of the Imperial Standards", London 1930 and 1936.

to express the lengths of the old Indian standards in modern metres, it therefore seems better to use Benoit's ratio and to rely on the stability of the yard between 1865 and 1894, rather than to use Sears' ratio and rely on stability for a longer period. The standard yard was cast in 1845, so the period 1865–1894 represents the 21st to 50th years of its age. Between its 21st and 50th years P.C. VI shortened by 1·2 in 10^6 of its length, so it is possible that the standard yard did the same, and the accepted values of the Indian standards may for this reason be in error by some such amount.

Table 13, column 3, gives the 1865 lengths of the Indian standards converted to metres by Benoit's 1894 ratio. It follows that:—

1 Indian foot = 0·333 331 89 yards of 1865–1894 = 0·304 798 41 international metres.

The acceptance of Benoit's 1894 ratio in preference to Sears' 1926 (which is generally accepted as correct) implies the belief that the standard yard shortened by 0·86 in 10^6 between 1894 and 1926*, so the following relation must be added to the above:—

1 Indian foot = 0·333 332 17 yards of 1926.

5. Comparisons of 1907.—Bars A, I_S and I_B still exist. In 1907–08 bar A was compared with I_S and I_B in India and with the metre at Sèvres†. The British Ordnance Survey bar 01_1 which Captain Clarke had compared in 1865, was also compared with the metre in 1906. The results of these comparisons are given in Table 13, column 4, which shows that bars A, I_S, I_B and 01_1 apparently increased by +3·9, +3·9, −1·3 and +2·3 in 10^6 respectively between 1865 and 1907. These changes are not very satisfactory, although not really bad. Except for I_B the agreement *inter se* is very good, and taken by itself might even suggest that the 1907 comparison with the metre was at fault. The changes may be due to either:—

(1) Errors in the values accepted for 1865

(2) Changes in the lengths of the bars between 1865 and 1907

(3) Errors in the 1907 comparisons.

As regards the last the 10-foot bars (Indian and British) were in 1907 compared with sub-standards G_1 and G_2 at Sèvres. The comparison of these sub-standards with the metre is described in the Ordnance Survey Professional Paper (New Series) No. 1‡. The work carries the authority of the Bureau International, and must be of high quality, although a comparison between feet and metres is a more difficult operation than an ordinary comparison between commensurable lengths.

As regards change in the lengths of the Indian bars between 1865 and 1907, this is regrettably possible. There was a time, probably about 1885, when they were regarded as obsolete. They were stored in Calcutta, and it has even

* This may not be the case, but it seems more probable than the assumption that the yard changed by a smaller amount between 1865 and 1926. If the yard has been stable during 1894–1926 and if Sears' ratio is correct, Benoits' ratio was wrong by 0·86 in 10^{-6} in 1894. But if the yard shortened by about 1 in 10^6 during 1865–1894 as is quite probable, Benoits' ratio would be very accurately applicable to its 1865 value, which is what has now been assumed.

† Survey of India Departmental Paper No. 7, by Major H. M. Cowie, R.E.

‡ Figures are there given for the comparisons of G_1 and G_2 with new marks on the sides of 01_1. The results of comparison with the old centre-line marks which had been used by Clarke have been communicated in a letter from the Director of the Bureau, dated 24-12-37. Geodetic Branch file 47-R of 1938.

been said that for some time they stood in a vertical position*. This may not be true, but it is certainly possible that they may have suffered from treatment which would account for all the change. For at least the last 30 years bar A itself has been very slightly bent.†

It has been thought best to ignore the 1907 comparisons, except in so far as they do provide an approximate check on the accuracy of the old work, and as stated above reliance is placed on Clarke's and Benoit's comparisons and on the stability of the yard between 1865 and 1894. It is thought that the accepted value of bar A probably differs by at least 1 in 10^6 from its true value in 1867, but that an error of more than 3 in 10^6 is very unlikely.

6. Modern base-lines.—The invar wires used for modern base-lines have been standardized by comparison with a nickel and a silica metre, whose lengths in terms of the International metre were determined in 1931 at the National Physical Laboratory. See Geodetic Reports Vols. VII, 1933 and 1934. The resulting lengths of the new base-lines have been reduced to Indian feet by the relation given in para 4, namely 1 Indian foot = 0·304 798 41 metres. The error of standard common to all the wires used is not likely to exceed 1, or possibly 1½, in 10^6 (See Geodetic Report 1934, page 13).

TABLE 13.—*Standards of length at 62° F.*

(1) Standard	(2) In yards Clarke 1865	(3) In metres by Col. (2) and Benoit 1894	(4) In metres 1906–08
Indian 10-foot Bar A ...	3·333 318 86	3·047 984 1	3·047 996
„ Bar I_s ...	3·333 401 38	3·048 060	3·048 072
„ Bar I_D ...	3·333 532 84	3·048 180	3·048 176
Ordnance Survey 10-foot Bar 01_1 (centre marks) ...	3·333 354 32	3·048 016	3·048 023
Standard yard of 1865–94 ...	1·000 000 00	0·914 399 2	...

* Communicated by Sir Sidney Burrard in a letter dated 16-7-37. Geodetic Branch file 47-R of 1937. Sir Sidney joined the Survey of India in 1884, and remembers hearing that this may have happened.

† Not by such an amount that the length along the curve differs appreciably from that along the chord, but it may be a sign of ill-treatment and consequent change.

Appendix XI

THE DEFINITION OF THE REFERENCE SPHEROID

1. Definitions necessary.—The definition of any spheroid requires seven constants. The constants of a spheroid used for the computation of triangulation are usually defined as follows:—

(*a*) The minor axis of the spheroid is defined to be parallel to the earth's axis of rotation. This determines two constants, which are common to the spheroids used in all countries.

(*b*) Lengths are assigned to the major and minor axes, or to the major axis and the flattening. Different countries have used different values, but "International" values have now been agreed on, which are suitable for universal use.

(*c*) It remains to assign a position to the centre of the spheroid, and so to determine the remaining three constants. It would be possible to define the centre to be at the earth's centre of gravity. If all countries did this, and also adopted the International axes, it would have the happy result of placing all surveys on the same spheroid. Unfortunately this is not possible, for the surveyor standing on his triangulation station has no precise knowledge of the direction or distance of the earth's centre of gravity, and he cannot relate his measures to it. He is therefore forced to adopt the following procedure, which allows him to compute his observations accurately, but places all independent surveys on spheroids with different centres.

At one station known as the origin the surveyor arbitrarily defines the spheroidal or geodetic latitude and longitude*, and also the height of the spheroid above or below the geoid. Astronomical observations tell him the true or astronomical latitude and longitude*, and the differences between these and the arbitrary geodetic values tell him the angles between the spheroidal normal and the direction of gravity at the station. Spirit levelling tells him the height of his station above the geoid, and hence his definition gives its height above the spheroid. At his origin he has thus defined the distance between ground level and the spheroid, and also the direction of the spheroidal normal at this point, and so has defined the remaining three constants.

At the origin the geodetic and astronomical latitude and longitude are often defined to be equal, so that geoid and spheroid are parallel, but this is not necessary. The necessity for defining the separation between geoid and spheroid has generally been ignored, but as base-lines are generally reduced to sea-level through geoidal instead of spheroidal heights, the separation is implied to be about zero.

2. Everest's spheroid.—The definition of the Indian (Everest's) spheroid is as follows.

Semi major axis, a = 20,922,931·80 feet.

Semi minor axis, b = 20,853,374·58 feet.

Whence Flattening = 1 : 300·8017.

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* Or alternatively the latitude, and the azimuth of an adjacent station.

At Kaliānpur Hill station the geodetic latitude is defined to be 24° 07′ 11″·26, and the azimuth of Sūrantāl is defined to be 190° 27′ 5″·10. From these values it is deduced that the inward normal to the geoid lies 0″·29 north and 2″·89 west of the inward spheroidal normal. At Kaliānpur the height of the geoid above Everest's spheroid is defined to be zero.

These deflections arise from an astronomical value of the latitude of 24° 07′ 10″·97, which is the mean of many observations (see Professional Paper No. 5, page 5), and from an astronomical azimuth of 190° 27′ 06″·39. The 1899 value of the azimuth was 190° 27′ 06″·37, but it is convenient to accept the insensibly different 06″·39 for the reasons given in Geodetic Report Vol. VI Supplement, page viii footnote. For longitude see para 4 of this Appendix.

Of late years the definition of Everest's spheroid has been made with reference to a fictitious Kaliānpur Origin rather than to Kaliānpur H.S. See Professional Paper No. 5 and Geodetic Report Vol. VI Supplement, pages vi to viii. The history of this Origin is that in 1899 the deviation of the vertical was observed at a group of stations surrounding Kaliānpur, and it was thought that the mean of all would be more free from local disturbance of the vertical than Kaliānpur itself would be. It has since been realized that the mean of a small group is little more typical of the whole country than is a single station, and it is thought that the use of this group as a composite Origin has no advantage and may lead to difficulty and misunderstanding. The definition now given therefore takes the form of the deviations at Kaliānpur H.S. itself. The values given here and in para 5 correspond to the group means currently quoted, namely 0″·31 south and 2″·89 west for Everest's spheroid, and 3″·02 south and 3″·17 west for the International. There are no changes in the spheroids defined, but the definitions have been simplified.

3. **Unit of length.**—Colonel Everest defined the axes of his spheroid as above, and these figures have been retained unchanged in all computations. The unit of length which has been used, however, is not the British foot, but the Indian foot or tenth part of Standard Bar A as it was in 1831–82 when the base-lines were being measured, and it follows that the axes of the spheroid which has actually been used contain the specified number of Indian feet, not of British feet. The length of the Indian foot has been discussed in Appendix X, and from the figures there given the dimensions of the spheroid actually used may be derived as follows :—

Semi major Axis	Semi minor Axis	
20,922,841	20,853,284	British feet of 1865–94
20,922,859	20,853,302	British feet of 1926
6,377,276	6,356,075	International metres

It would be wrong to ascribe any of these figures to "Everest's spheroid", for it was defined as in para 2, and the figures there given are those used in all Tables prepared for Everest's spheroid. Everest's spheroid has, moreover, been used by countries, such as Siam, which have no connection with the Indian foot. It is thought that the best way of viewing this complication without confusion is to regard the original definition of any spheroid as giving the length of its axes in so many *units*, not necessarily exactly feet or metres, while the length of the unit used is derived from a consideration of the fundamental standard of the country concerned. The semi major axis of Everest's spheroid then contains 20,922,931·80 units as defined, while the Indian Survey has used *an* Everest spheroid in which the unit is the Indian foot.

4. Error in longitude.—An unfortunate confusion exists in connection with Indian longitudes. The angle between the geoidal and spheroidal normal at Kaliānpur H.S. is defined to be $2''\cdot 89$ as given in para 2, and this accords with the difference of $1''\cdot 29$ between the azimuth of the adjacent Sūrantāl H.S. as observed astronomically and as accepted for the geodetic computations. Unfortunately, in 1905 when modern longitudes were introduced, the geodetic longitude of Kaliānpur was taken to be equal to the observed astronomical value, instead of $3''\cdot 16$ less, which would have accorded with the deviation of $2''\cdot 89$ west. See Geodetic Report Vol. VI, Supplement, pages vii–ix. It is out of the question to correct either the azimuths or the longitudes of all Indian stations now, and the inconsistency must remain until the triangulation is eventually converted to a modern spheroid. The longitudes will then be corrected, see Chapter III, end of para 25. In the mean time, allowance for it is being made by using the formula $\xi = (A - G + 3''\cdot 16) \cos\phi$ when deducing the deviation of the vertical from longitude observations, and modifying Laplace's equation in a similar way*.

5. International spheroid.—The axes of the International spheroid are defined to be :—

$$a = 6{,}378{,}388 \text{ metres} = 20{,}926{,}506 \text{ British feet (of 1926).}$$

$$\text{Flattening} = 1:297\cdot 0.$$

$$\text{whence } b = 6{,}356{,}912 \text{ metres} = 20{,}856{,}047 \text{ British feet (of 1926).}$$

It is not possible to give anything that can be described as International values of the deviation at the Kaliānpur H.S., but in 1927 it was found that the values $2''\cdot 42$ south and $3''\cdot 17$ west procured the best fit between a spheroid with International axes and the compensated geoid as then known. The same consideration gave the height of the geoid above the spheroid at Kaliānpur to be 31 feet. Knowledge of the form of the geoid in India has since increased, but these values are still quite appropriate. It is clear, however, that some great extension of knowledge, such as would be derived from a connection with European surveys, might introduce considerable changes.

Using the notation of Chapter III, para 23, the difference between the Indian Everest's and International spheroids are (International *minus* Everest. Both expressed in British feet of 1926) :—

$$\delta a\dagger = +\ 3647 \text{ feet}$$

$$\delta\epsilon = +\ 0\cdot 4255 \times 10^{-4}$$

$$N_0 = -\ 31 \text{ feet}$$

$$\delta\eta_0 = +\ 2''\cdot 71 \text{ (south } \eta \text{ is } +^{ve})$$

$$\delta\xi_0 = +\ 0''\cdot 28 \text{ (west } \xi \text{ is } +^{ve})$$

---

* Values of the deviation published in the Supplement to Geodetic Report Vol. VI, and other Geodetic Reports have been computed in this way, and published geoid charts are free from error on this account.

† If the axes of the International Spheroid are to be expressed in British feet of 1926, the sides of the triangulation must be in the same units. The log sides now expressed in Indian feet must therefore receive corrections of $-0\cdot 000.0015$ before conversion to the International spheroid by the formula of para 23.

Appendix XII

CONNECTION BETWEEN A TRIANGULATION STATION AND THE CORRESPONDING POINT ON THE SPHEROID

If A is a triangulation station it is necessary to define precisely the connection between it and A′, the corresponding point on the spheroid. See para 21. It is possible to define A′ as being the point vertically below A, in the sense that a body at A is urged by gravity to follow the path AA′. It is more convenient, however, to define A′ as the point from which the spheroidal normal passes through A, and a little more convenient still to define A′ so that AA′ is slightly curved. It is normal to the spheroid at A′ and curved (concave to the polar axis) in the meridian plane with a radius of curvature of $\rho/0 \cdot 0053 \sin 2\phi$*.

With this definition, the corrections necessary to convert an observed angle to a spheroidal angle have been given by Dr. J. de Graaff Hunter in Departmental Paper No. 12, Section II (4). Of the various corrections involved, by far the largest is that due to the deviation of the vertical, and as this is generally unknown and neglected the other corrections can also be ignored. The correction to an observed angle ASB on this account is $(\xi \cos \beta_A - \eta \sin \beta_A) \tan \alpha_A - (\xi \cos \beta_B - \eta \sin \beta_B) \tan \alpha_B$, where β_A and β_B indicate azimuths at S measured clockwise from south, while α_A and α_B indicate angles of elevation at S. The sign will be correct if the stations of the triangle are lettered BSA in clockwise order.

* This curvature is the same as that of the path along which a body is urged by gravitation, if the latter is undisturbed by gravitational anomalies. It is orthogonal to the (undisturbed) equipotential surfaces.

Appendix XIII

EXAMPLE OF NEGLECTING DEVIATION CORRECTIONS

Suppose that a series AB......A_n B_n (Plate XIII) has been computed on an ill-fitting spheroid S_1 such as Everest's, and that the deviation corrections have been neglected. It is now required to convert it to a new spheroid S_2 such as the International, which fits the geoid well. Let AB and A_nB_n be two measured base-lines, and let A and A_n be two Laplace stations, all of which have been correctly reduced to the spheroid S_1 (Everest's). In the area concerned let the mean value of the components of the fairly constant angles between the two spheroids be η_m and ξ_m. Random deviations of the vertical, different at all stations, also occur but are unknown. They have been neglected on S_1 and will be neglected on S_2. For simplicity let the series consist of equilateral triangles of side l miles, and at first let all the stations be at sea-level, so that all vertical angles are depressions of $l/2a$, where a is the earth's radius.

The errors arising from the neglect of η_m and ξ_m may be considered under two heads as follows:—

(1) Errors due to neglect of ψ, the component of η_m and ξ_m lying along the series. The correction which should have been applied to any angle is given by Appendix XII, and it is easily seen that if these corrections had been applied in the computations on S_1, the series would have taken the form shown in broken lines in Plate XIII, figure 1. The change there shown is simply a progressive change of scale, and the change at any point A_n is $L\psi/a$, where L is the distance AA_n.

Then if A_nB_n is a measured base-line which has been properly reduced to spheroid level on S_1, the scale of the triangulation after adjustment on the base-line will be everywhere correct, provided only that the closing error is properly distributed*.

(2) Errors due to neglect of χ, the component of η_m and ξ_m lying across the series. As in (1) the necessary corrections to each angle are obtainable from Appendix XII, and if they had been included in the computations on S_1, the series would have taken the form shown in broken lines in Plate XIII, figure 2. In this case the change is a progressive swing in direction which amounts to $L\chi/a$ radians after a distance L. This similarly is eliminated by adjustment on Laplace stations.

Provided the scale and azimuth of the series is controlled by fairly frequent base-lines and Laplace stations, the scale and azimuth of each side is thus correct in spite of neglect of the deviation corrections, and the whole series is correctly computed on the spheroid S_1. The formulæ of para 23 can then be used for its conversion to S_2.

Now let one station, say A_2, stand at a height h above sea-level, while the others are at sea-level as before. Ignoring the deviation corrections arising from the depressions $l/2a$, which have already been dealt with, Plate XIII

* It is shown in para 25 that the usual distribution of closing error is adequate for this purpose.

figure 3 shows in broken lines the results of including deviation corrections due to this elevation of A_2. Scale, azimuth and positions beyond $A_3 B_3$ are unaffected, and the only change is in the position of A_2 itself and in the scale and azimuth of sides passing through it. It is easy to see that the change in the position of A_2 is $h\eta_m/a$ in latitude and $h\xi_m/a \sec\phi$ in longitude. To this extent, neglect of the deviation corrections has left the computations on S_1 in error even after adjustment on $A_n B_n$, but as these errors of position are exactly equal to the height terms in the formulæ of para 23, they are eliminated by neglect of the height terms when converting the triangulation to S_2. The errors of scale and azimuth in the sides passing through A_2 are similarly eliminated by neglecting the height terms in the formulæ for converting scale and azimuth.

It is clear that the above considerations continue to apply when other stations besides A_2 are raised to different heights above sea-level.

Effect of neglecting Deviation Corrections

Fig. 1

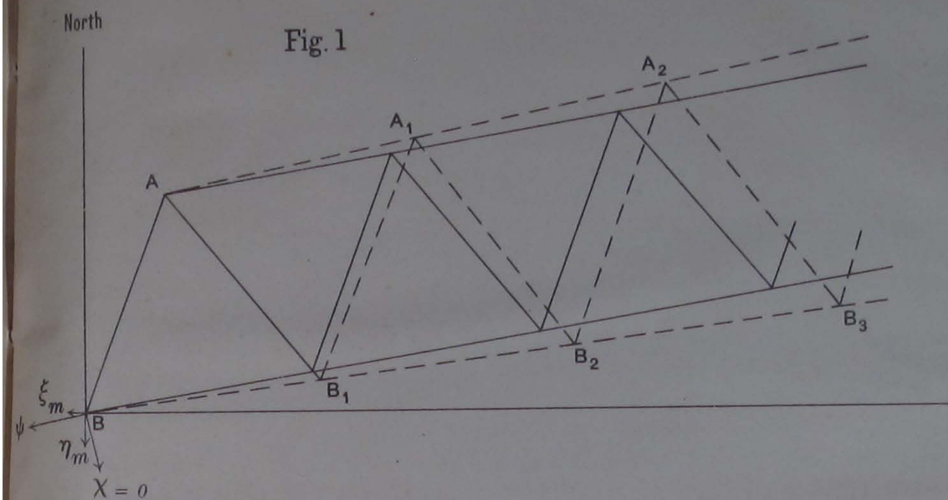


Fig. 2

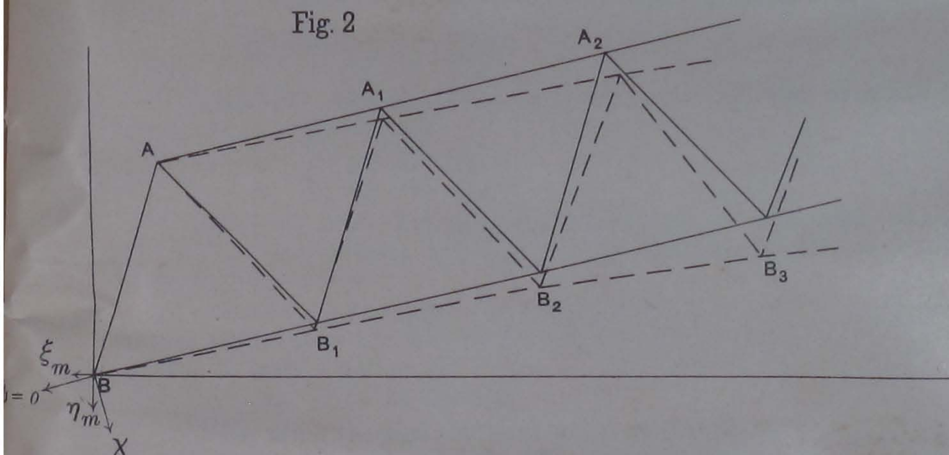
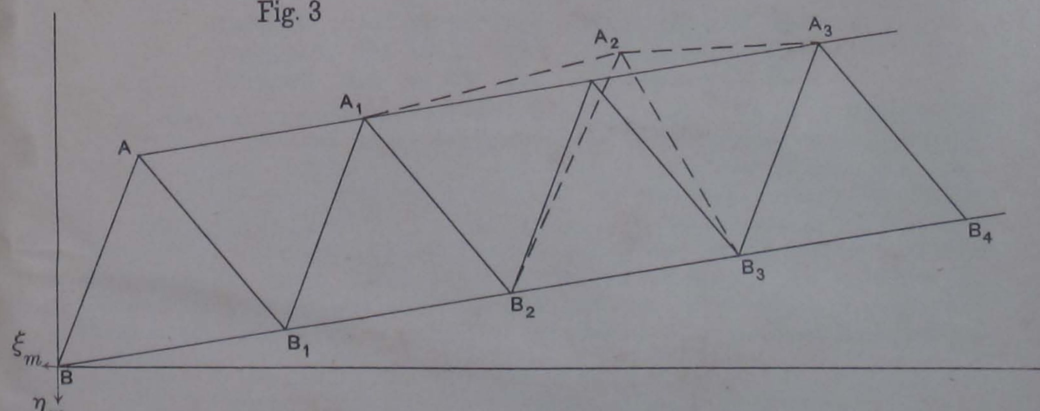


Fig. 3



Full lines represent computation on Everest's spheroid neglecting deviation corrections.

Broken lines indicate the effect of including corrections arising from constant deviations η_m and ξ_m.

All stations are at sea-level except A_2 in Fig. 3.

Appendix XIV

EXAMPLE OF CONVERSION TO THE INTERNATIONAL SPHEROID

Let A be a point 16° 00′ 00″·000, 96° 00′ 00″·000 on Everest's spheroid. Let the azimuth of another point B be 35° 00′ 00″·00, and let log AB = 5·300 0000 (Indian) feet. Then the position of B as computed by azimuth and distance on form 13 A Trian. is 15° 32′ 58″·282, 95° 40′ 29″·208, and the reverse azimuth is 214° 54′ 41″·71. Let both A and B be at sea-level.

Let these azimuths, distance and positions now be converted to the International spheroid by the formulæ of Chapter III para 23, using the values of δa, $\delta \epsilon$ etc. given in Appendix XI.

Latitude. The formula gives $\delta\phi = +0''\cdot914$ and $+1''\cdot047$ at A and B respectively, and their latitudes on the International spheroid are then

A. 16° 00′ 00″·914
B. 15° 32′ 59″·329

Longitude. Similarly $\delta\lambda = -11''\cdot915$ and $-11''\cdot689$ at A and B, and International longitudes are

A. 95° 59′ 48″·085 *
B. 95° 40′ 17″·519 *

Log side. Plate XII gives a correction of +0·000 0028, whence log AB on the International spheroid is 5·300 0028 Indian feet or 5·300 0013 British feet of 1926 †.

Azimuth. Corrections δA are $-3''\cdot28$ and $-3''\cdot13$ at A and B‡, whence azimuths on the International spheroid are

AB. 34° 59′ 56″·72
BA. 214° 54′ 38″·58

Now, using the International values of the latitude and longitude of A and of the side and azimuth AB as obtained above, let the latitude and longitude of B be computed on form 13 A Trian§.

The result is

Latitude	15°	32′	59″·327.	Discrepancy 0″·002
Longitude	95	40	17 ·518.	Discrepancy 0 ·001
Azimuth	214	54	38 ·49.	Discrepancy 0 ·09

The discrepancies are all satisfactorily small. Those in latitude and longitude are such as would naturally arise in the course of computation. That in azimuth, which is equally negligible, is easily seen to arise from neglect of the change in deviation correction. At each station the vertical angle is a depression of about 20′, and the change in deviation (across the line AB) is about 10″. So each azimuth requires a correction of 10″ tan 20′ or 0″·05, with opposite signs, which explains the discrepancy. As explained in Chapter III para 25, and Appendix XIII, the cumulative effect of neglecting these small corrections is remedied by closure on base-lines and Laplace stations.

* 3″·16 is also to be subtracted. See last sub-para of Chapter III. para 25.

† See Appendix XI, para 5.

‡ These are the exact figures from computation. From Plate XII they can be read to one decimal only.

§ The values of ρ and ν in feet must be obtained from the International metre Tables by the relation 1 metre = 39·370147 British inches of 1926, since this ratio accords with the correction of −0·000 0015 to convert Indian log feet to British.

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